Precalculus Guided Notes: INTRO to Limits

Name ____ Date ____________

Review Questions

1) Solve for all values of x $\cos^2 x - 2\cos x + 1 = 0$
2) The graph of $r = 3\cos\theta + 3$ is a ______________ with an extent of _____, dotted lines at θ = __________. This is where r = 0.
3) Given SAS, use Law of __________________.

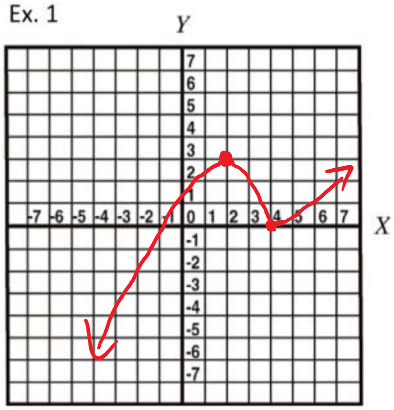
LIMIT: A limit represents a value that a function is approaching at a specific x value

In general, $\lim_{x \to \infty} f(x) = L$ Read as "the limit of f(x) as x approaches a equals L"

This means that as x gets closer to a, but remains unequal to a, the corresponding values of f(x) get closer to L.

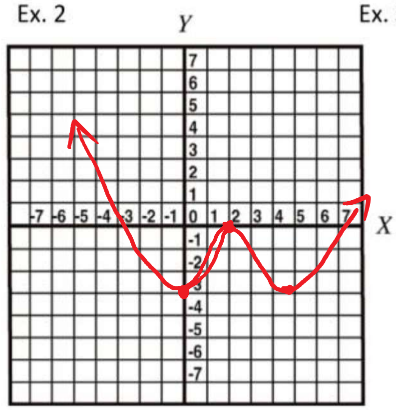
Use the graph of f to find the indicated limit and function value.

Ex. 1



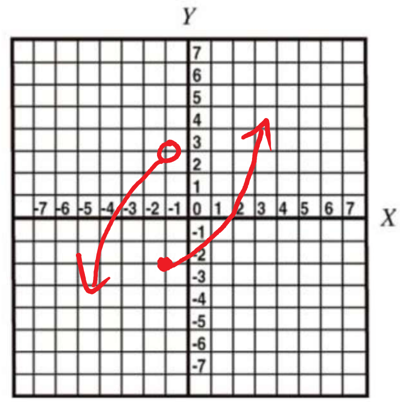
lim f(x) f(2)

Ex. 2



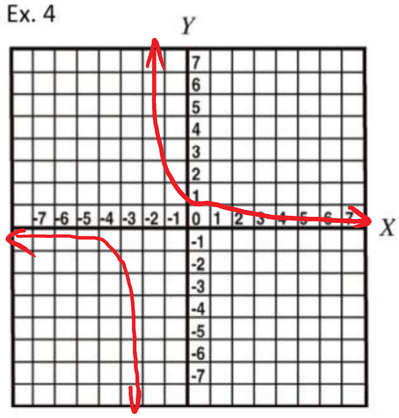
limf(x) f(0)

Ex. 3



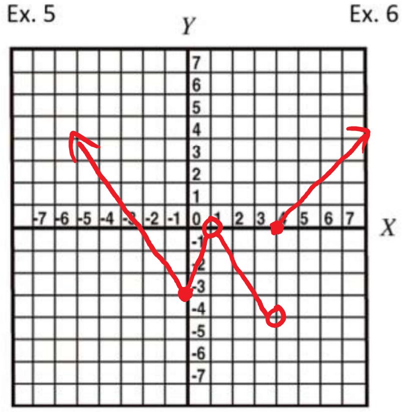
limf(x) f(-1)

Ex. 4



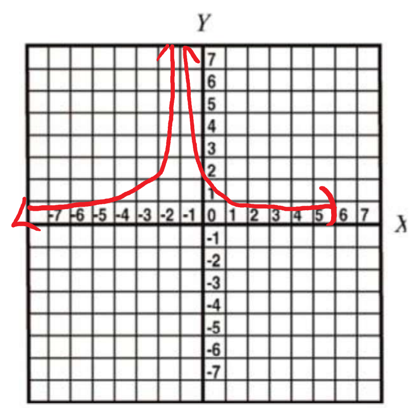
limf(x) f(-2)

Ex. 5



lim(fx) f(1)

Ex. 6



limf(x) f(-1)

Precalculus Guided Notes: INTRO to Limits

Construct a table to find the limit of each. Think of the graph mentally as well to help with finding the limit.

Ex. 7 $\lim_{x \to 0} (5x)$

Ex. 8 $\lim_{x \to -1^+} 1/(x+1)$

$x \to -1-$

$x \to -1$

Ex. 9 $\lim_{x \to 0} \tan x / x$

Ex. 10 $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x + 1 & \text{if } x < 1 \\ 3x - 3 & \text{if } x \geq 1 \end{cases}$

Limits: If the limit as x approaches a from the right = the limit as x approaches a from the left then the limit as x approaches a exists.

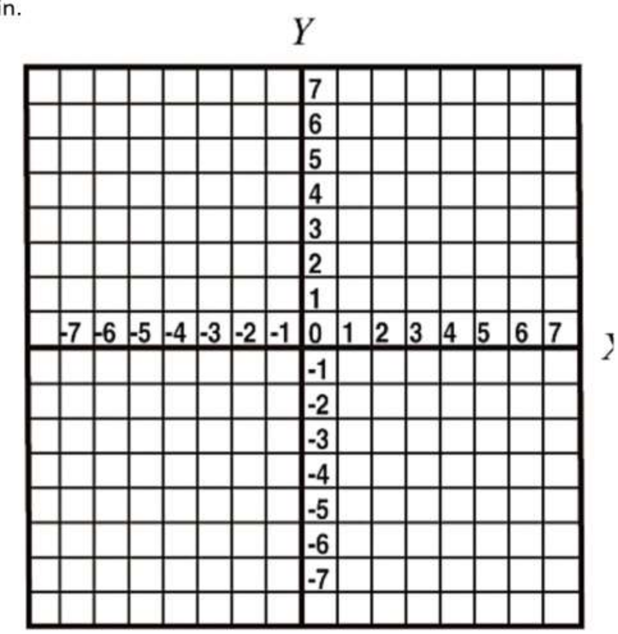
If these two limits (from the left and from the right) are not equal then the limit does not exist.
Strategies

- Make a graph of the function.
- Make a chart of values approaching the number a.
- If you know the graph is continuous then just plug in.

Ex. 11

$$f(x) = \begin{cases} 1 & x < -1 \\ 2 & x = -1 \\ -x & -1 < x \leq 0 \\ x + 2 & 0 < x < 1 \\ 1 & x = 1 \\ -4x + 7 & 1 < x < 2 \\ -1 & x \geq 2 \end{cases}$$

lim f(x)	lim f(x)	limf(x)	f(-1) =
lim f(x)	lim f(x)	limf(x)	f(0) =
lim f(x)	lim f(x)	limf(x)	f(1) =
lim f(x)	lim f(x)	limf(x)	f(2) =



11.2 Finding Limits

Pre-Calculus 11.2 Finding Limits using the Properties

Review

Given $\boldsymbol{a} = 5i + 2j$, $\boldsymbol{b} = 4i - 5j$, find

1. *Find the direction of vector a.*
2. *Find the magnitude of vector b.*
3. Find $\|\boldsymbol{a} + \boldsymbol{b}\|$

Objective: Finding limits using the properties of limits

$\lim_{x \to a} x = a$

$\lim_{x \to a} c = c$

Ex 1) $\lim_{x \to 5} (12 + x)$

Ex 2) $\lim_{x \to 2} (2x - 5)(x + 3)$

Ex 3) $\lim_{x \to -2} \sqrt{4x^2 + 5}$

Ex 4) $\lim_{x \to 3} (4x^3 + 2x^2 - 6x + 5)$

Ex 5) $\lim_{x\to -2} \frac{3x}{x-4}$

Ex 6) $\lim_{x\to 3} \frac{x^2-x-6}{x^2-9}$

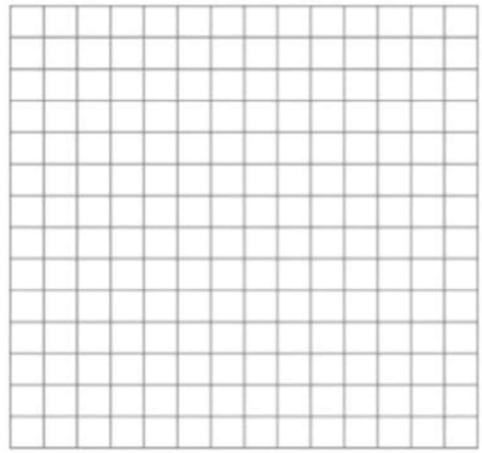
Ex 7) $\lim_{x\to 5} \frac{2x^2-7x-15}{x-5}$

Ex 8) $\lim_{x\to 0} \frac{\sqrt{16+x}-4}{x}$

Ex 9) $\lim_{x\to 9} \frac{\sqrt{x}-3}{x-9}$

Ex 10) $\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

Ex 11) $f(x) = \begin{cases} x^2 + 5 \; if \; x < 2 \\ 3x + 1 \; if \; x \geq 2 \end{cases}$



a. $\lim_{x \to 2^-} f(x)$

b. $\lim_{x \to 2^+} f(x)$

c) $\lim_{x \to 2} f(x)$

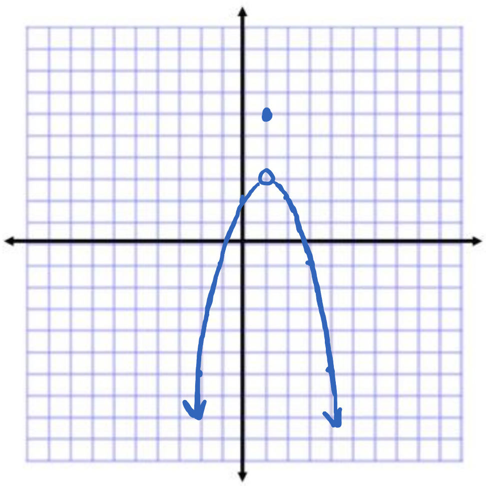
Ex 12) $f(x) = \begin{cases} x^2 + 6 \; if \; x < 2 \\ x^3 + 2 \; if \; x \geq 2 \end{cases}$

Ex 13) $f(x) = \begin{cases} \frac{x^3 - 64}{x - 4} \; if \; x \neq 4 \\ 7 \; if \; x = 4 \end{cases}$

11.3 Limits and Continuity Temp

Goal: Determine if a function is continuous at a specific number and determine where functions are discontinuous.

Review Questions: Use the graph at the right:



1) $\lim_{x\to 1^+} f(x) =$

2) $\lim_{x\to 1^-} f(x) =$

3) $\lim_{x\to 1} f(x) =$

4) $f(1) =$

Definition of a Function Continuous at Number:

1) f is defined at a, that is, a is in the domain of f, so that $f(a)$ is a real number.

2) $\lim_{x\to a} f(x)$ exists.

3) $\lim_{x\to a} f(x) = f(a)$

Determine if the following functions $f(x)$, are continuous at the given value a.

Ex. 1 $f(x) = \frac{x-2}{x^2-4}$ $for\ a = 1\ and\ for\ a = 2$

Ex. 2 $f(x) = \begin{cases} \frac{x^2-16}{x+4} & if\ x \neq -4 \\ 5 & if\ x = -4 \end{cases}$ $for\ a = -4$

Ex. 3 $f(x) = \begin{cases} x - 2 & if\ x \leq 0 \\ 3x^2 - 3x - 2 & if\ x > 0 \end{cases}$ $for\ a = 0$

Determine for what numbers, if any, the given function is discontinuous.

Ex. 4 $f(x) = \frac{x-7}{x^2-5x-14}$

Ex. 5 $f(x) = \frac{tanx}{x}$

Ex. 6 $f(x) = 10$

Ex. 7 $f(x) = \cot x$

Ex. 8 $f(x) = 3\log(x)$

.

Ex. 9 $f(x) = 3\ln(x+ 5)$

Ex.10 $f(x) = \sin(x)/(x + 6)$

11.3A - Limits and Continuity Temp

Goal: Determine if a function is continuous at a specific number and determine where functions are discontinuous.

Review Questions: Determine where each function is discontinuous.

1) $f(x) = \dfrac{x-3}{x^2+4x-21}$

2) $f(x) = \dfrac{cosx}{x}$

3) $f(x) = \sqrt{2}$

Recall - Definition of a Function Continuous at Number:

1) f is defined at a, that is, a is in the domain of f, so that $f(a)$ is a real number.

2) $\lim\limits_{x\to a} f(x)$ exists.

3) $\lim\limits_{x\to a} f(x) = f(a)$

Determine if the following function $f(x)$, is continuous at the given value a.

Ex. 1 $f(x) = \begin{cases} \frac{x^4-16}{x-2} & if\ x \neq 2 \\ 32 & if\ x = 2 \end{cases}$ $for\ a = 2$

Ex. 2 $f(x) = \begin{cases} \sqrt{3-x} & if\ x \le 3 \\ x^2 - 3x & if\ x > 3 \end{cases}\ for\ a = 3$

Determine for what numbers, if any, the given function is discontinuous.

Ex. 3 $f(x) = \begin{cases} 2x & if\ x \le 0 \\ x^2 + 1 & if\ 0 < x \le 2 \\ 7 - x & if\ x > 2 \end{cases}$

Ex. 4 $f(x) = \begin{cases} \frac{sin3x}{x} & if\ x \ne 0 \\ 3 & if\ x = 0 \end{cases}$

Ex. 5 $f(x) = \begin{cases} x + 1 & if\ x \le 3 \\ x^2 - 3 & if\ x > 3 \end{cases}$

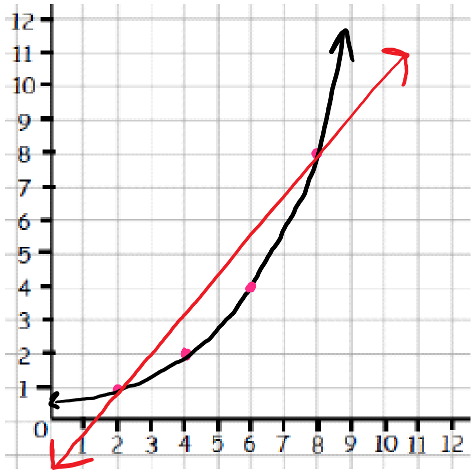
11.4 - Introduction to Derivatives

Goal: Find the slope of a tangent line at a given point in a function and write a slope intercept equation of the tangent line.

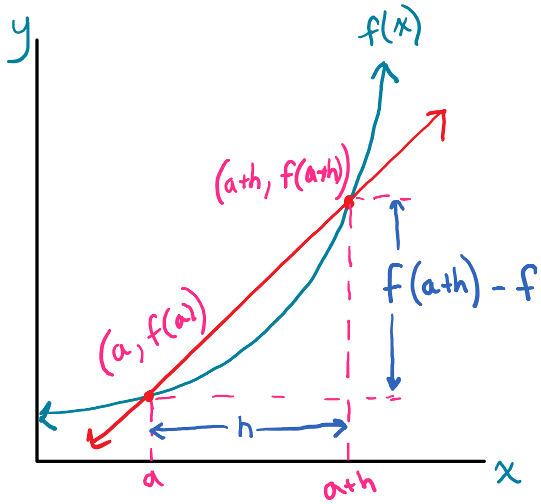
Review Question: Determine for what numbers, if any, the following function is discontinuous.

1) $f(x) = \begin{cases} x^2 & if\ x < -2 \\ -x+2 & if\ -2 \le x < 1 \\ 3x & if\ x \ge 1 \end{cases}$

Consider the following graph:



How do we find the slope of the line that connect the points (2,1) and (8,8)? This is called a secant line. What changes if we connect from (2,1) to (6,4)? From (2,1) to (4,2)? What if I want to find the slope at the single point (2,1)?



Consider the situation in general terms…

slope =

To find the slope at a single point (tangent line), let the horizontal distance go to zero or find the limit as *h* approaches zero.

$$\lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$

This is also called the instantaneous rate of change of *f* with respect to *x* at *a.*

Ex. 1   Find the slope of the tangent line to the graph of $f(x) = x^2 + x$ at $(2,6)$. Then find the slope-intercept equation of the tangent line.

Ex. 2   Find the slope of the tangent line to the graph of $f(x) = 2x^2 + x - 2$ at $(1,1)$. Then find the slope-intercept equation of the tangent line.

Ex. 3   Find the slope of the tangent line to the graph of $f(x) = \sqrt{x}$ at $(4,2)$. Then find the slope-intercept equation of the tangent line.

Ex. 4   Find the slope of the tangent line to the graph of $f(x) = \frac{3}{x}$ at $(1,3)$. Then find the slope-intercept equation of the tangent line.

11.4A - Introduction to Derivatives Continued

Goal: Find the derivative of a function and use it to find the slope of tangent lines.

Review Question: Find the slope of the tangent line to the graph of $f(x) = x^3$ at (2,8).

Definition of the Derivative of a Function:
For a function $f(x)$, the derivative of f at x, denoted by $f'(x)$ read "f prime of x" is defined by

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

provided that this limit exists. The derivative of a function gives the slope of the function for any value of x in the domain of $f'(x)$.

Ex. 1 Find the derivative of $f(x) = x^2 + 3x \; at \; x$. Then find the slopeof the tangent line of $f(x)$ at $x = -2 \; and \; x = -\frac{3}{2}$.

Ex. 2 Find $f'(x)$ of $f(x) = x^3 - 2\ at\ x$. Then find the slopeof the tangent line of $f(x)$ at $x = -1\ \ and\ x = 1$.

Ex. 3 Find the derivative of $f(x) = \sqrt{x} + 2\ at\ x$. Then find the slopeof the tangent line of $f(x)$ at $x = 4\ \ and\ x = 1$.

Ex. 4 Find $f'(x)$ of $f(x) = \frac{8}{x}\ at\ x$. Then find the slopeof the tangent line of $f(x)$ at $x = -2\ \ and\ x = 1$.