4.4A Homework Answers

Thursday, October 19, 2017 6:48 AM

7. We need values for x, y, and r. Because $P = (-2, -5)$ is a point on the terminal side of θ, $x = -2$ and $y = -5$. Furthermore,

$r = \sqrt{x^2 + y^2} = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4+25} = \sqrt{29}$ Now that we know x, y, and r, we can find the six trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-5}{\sqrt{29}} = \frac{-5}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}} = -\frac{5\sqrt{29}}{29}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{29}} = \frac{-2}{\sqrt{29}}\cdot\frac{\sqrt{29}}{\sqrt{29}} = -\frac{2\sqrt{29}}{29}$$

$$\tan\theta = \frac{y}{x} = \frac{-5}{-2} = \frac{5}{2}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{29}}{-5} = -\frac{\sqrt{29}}{5}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{29}}{-2} = -\frac{\sqrt{29}}{2}$$

$$\cot\theta = \frac{x}{y} = \frac{-2}{-5} = \frac{2}{5}$$

23. In quadrant III x is negative and y is negative. Thus,

$\cos\theta = -\frac{3}{5} = \frac{x}{r} = \frac{-3}{5}$, $x = -3$, $r = 5$. Furthermore,

$$r^2 = x^2 + y^2$$
$$5^2 = (-3)^2 + y^2$$
$$y^2 = 25 - 9 = 16$$
$$y = -\sqrt{16} = -4$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-4}{5} = -\frac{4}{5}$$

$$\tan\theta = \frac{y}{x} = \frac{-4}{-3} = \frac{4}{3}$$

$$\csc\theta = \frac{r}{y} = \frac{5}{-4} = -\frac{5}{4}$$

$$\sec\theta = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{-4} = \frac{3}{4}$$

27. Because $270° < \theta < 360°$, θ is in quadrant IV. In quadrant IV x is positive and y is negative. Thus,

$\cos\theta = \frac{8}{17} = \frac{x}{r}$, $x = 8$,

$r = 17$. Furthermore

$$x^2 + y^2 = r^2$$
$$8^2 + y^2 = 17^2$$
$$y^2 = 289 - 64 = 225$$
$$y = -\sqrt{225} = -15$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$$

$$\tan\theta = \frac{y}{x} = \frac{-15}{8} = -\frac{15}{8}$$

$$\csc\theta = \frac{r}{y} = \frac{17}{-15} = -\frac{17}{15}$$

$$\sec\theta = \frac{r}{x} = \frac{17}{8}$$

$$\cot\theta = \frac{x}{y} = \frac{8}{-15} = -\frac{8}{15}$$

29. Because the tangent is negative and the sine is positive, θ lies in quadrant II. In quadrant II, x is negative and y is positive. Thus,

$\tan\theta = -\frac{2}{3} = \frac{y}{x} = \frac{2}{-3}$, $x = -3$, $y = 2$. Furthermore,

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + 2^2} = \sqrt{9+4} = \sqrt{13}$$

Now that we know x, y, and r, we can find the remaining trigonometric functions of θ.

$$\sin\theta = \frac{y}{r} = \frac{2}{\sqrt{13}} = \frac{2}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = \frac{2\sqrt{13}}{13}$$

$$\cos\theta = \frac{x}{r} = \frac{-3}{\sqrt{13}} = \frac{-3}{\sqrt{13}}\cdot\frac{\sqrt{13}}{\sqrt{13}} = -\frac{3\sqrt{13}}{13}$$

$$\csc\theta = \frac{r}{y} = \frac{\sqrt{13}}{2}$$

$$\sec\theta = \frac{r}{x} = \frac{\sqrt{13}}{-3} = -\frac{\sqrt{13}}{3}$$

$$\cot\theta = \frac{x}{y} = \frac{-3}{2} = -\frac{3}{2}$$

87. $\sin\frac{\pi}{\ }\cos\pi - \cos\frac{\pi}{\ }\sin\frac{3\pi}{\ }$

$\frac{17\pi}{\ } \quad \frac{5\pi}{\ } \quad \frac{17\pi}{\ } \quad \frac{5\pi}{\ }$

87. $\sin\frac{\pi}{3}\cos\pi-\cos\frac{\pi}{3}\sin\frac{3\pi}{2}$

$=\left(\frac{\sqrt{3}}{2}\right)(-1)-\left(\frac{1}{2}\right)(-1)$

$=-\frac{\sqrt{3}}{2}+\frac{1}{2}$

$=\frac{1-\sqrt{3}}{2}$

88. $\sin\frac{\pi}{4}\cos 0-\sin\frac{\pi}{6}\cos\pi$

$=\left(\frac{\sqrt{2}}{2}\right)(1)-\left(\frac{1}{2}\right)(-1)$

$=\frac{\sqrt{2}}{2}+\frac{1}{2}$

$=\frac{\sqrt{2}+1}{2}$

89. $\sin\frac{11\pi}{4}\cos\frac{5\pi}{6}+\cos\frac{11\pi}{4}\sin\frac{5\pi}{6}$

$=\left(\frac{\sqrt{2}}{2}\right)\left(-\frac{\sqrt{3}}{2}\right)+\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right)$

$=-\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

$=-\frac{\sqrt{6}+\sqrt{2}}{4}$

90. $\sin\frac{17\pi}{3}\cos\frac{5\pi}{4}+\cos\frac{17\pi}{3}\sin\frac{5\pi}{4}$

$=\left(-\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)$

$=\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}$

$=\frac{\sqrt{6}-\sqrt{2}}{4}$

91. $\sin\frac{3\pi}{2}\tan\left(-\frac{15\pi}{4}\right)-\cos\left(-\frac{5\pi}{3}\right)$

$=(-1)(1)-\left(\frac{1}{2}\right)$

$=-1-\frac{1}{2}$

$=-\frac{2}{2}-\frac{1}{2}$

$=-\frac{3}{2}$

92. $\sin\frac{3\pi}{2}\tan\left(-\frac{8\pi}{3}\right)+\cos\left(-\frac{5\pi}{6}\right)$

$=(-1)\left(\sqrt{3}\right)+\left(-\frac{\sqrt{3}}{2}\right)$

$=-\sqrt{3}-\frac{\sqrt{3}}{2}$

$=-\frac{2\sqrt{3}}{2}-\frac{\sqrt{3}}{2}$

$=-\frac{3\sqrt{3}}{2}$

99. $\sin\theta=\frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants I or II.

QI	QII
$\theta=\frac{\pi}{4}$	$\theta=\pi-\frac{\pi}{4}$
	$=\frac{3\pi}{4}$

$\theta=\frac{\pi}{4},\frac{3\pi}{4}$

100. $\cos\theta=\frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants I or IV.

QI	QIV
$\theta=\frac{\pi}{3}$	$\theta=2\pi-\frac{\pi}{3}$
	$=\frac{5\pi}{3}$

$\theta=\frac{\pi}{3},\frac{5\pi}{3}$

101. $\sin\theta=-\frac{\sqrt{2}}{2}$ when the reference angle is $\frac{\pi}{4}$ and θ is in quadrants III or IV.

QIII	QIV
$\theta=\pi+\frac{\pi}{4}$	$\theta=2\pi-\frac{\pi}{4}$
$=\frac{5\pi}{4}$	$=\frac{7\pi}{4}$

$\theta=\frac{5\pi}{4},\frac{7\pi}{4}$

4

$\theta = \frac{\pi}{4}, \frac{3\pi}{4}$

$\theta = \frac{5\pi}{4}, \frac{7\pi}{4}$

102. $\cos\theta = -\frac{1}{2}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or III.

QII	QIII
$\theta = \pi - \frac{\pi}{3}$	$\theta = \pi + \frac{\pi}{3}$
$= \frac{2\pi}{3}$	$= \frac{4\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{4\pi}{3}$

103. $\tan\theta = -\sqrt{3}$ when the reference angle is $\frac{\pi}{3}$ and θ is in quadrants II or IV.

QII	QIV
$\theta = \pi - \frac{\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3}$
$= \frac{2\pi}{3}$	$= \frac{5\pi}{3}$

$\theta = \frac{2\pi}{3}, \frac{5\pi}{3}$

104. $\tan\theta = -\frac{\sqrt{3}}{3}$ when the reference angle is $\frac{\pi}{6}$ and θ is in quadrants II or IV.

QII	QIV
$\theta = \pi - \frac{\pi}{6}$	$\theta = 2\pi - \frac{\pi}{6}$
$= \frac{5\pi}{6}$	$= \frac{11\pi}{6}$

$\theta = \frac{5\pi}{6}, \frac{11\pi}{6}$