5.1A HW Answers

Monday, November 20, 2017 12:09 PM

16. $\cos^2\theta(1+\tan^2\theta) = \cos^2\theta(\sec^2\theta)$

$$= \cos^2\theta \cdot \frac{1}{\cos^2\theta}$$

$$= 1$$

18. $\dfrac{1-\sin^2 t}{\sin t} = \dfrac{\cos^2 t}{\sin t}$

$$= \cos t \cdot \frac{\cos t}{\sin t}$$

$$= \cos t \cot t$$

20. $\dfrac{\sec^2 t}{\tan t} = \dfrac{\frac{1}{\cos^2 t}}{\frac{\sin t}{\cos t}}$

$$= \frac{1}{\cos^2 t} \div \frac{\sin t}{\cos t}$$

$$= \frac{1}{\cos^2 t} \cdot \frac{\cos t}{\sin t}$$

$$= \frac{1}{\cos t} \cdot \frac{1}{\sin t}$$

$$= \sec t \csc t$$

22. $\dfrac{\cot^2 t}{\csc t} = \dfrac{\csc^2 t - 1}{\csc t}$

$$= \frac{\csc^2 t}{\csc t} - \frac{1}{\csc t}$$

$$= \csc t - \sin t$$

24. $\dfrac{1-\sin\theta}{\cos\theta} = \dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}$

$$= \sec\theta - \tan\theta$$

26. $\dfrac{\sin t}{\tan t} + \dfrac{\cos t}{\cot t} = \dfrac{\sin t}{\frac{\sin t}{\cos t}} + \dfrac{\cos t}{\frac{\cos t}{\sin t}}$

$$= \sin t \div \frac{\sin t}{\cos t} + \cos t \div \frac{\cos t}{\sin t}$$

$$= \sin t \cdot \frac{\cos t}{\sin t} + \cos t \cdot \frac{\sin t}{\cos t}$$

$$= \cos t + \sin t$$

$$= \sin t + \cos t$$

28. $\cot t + \dfrac{\sin t}{1+\cos t} = \dfrac{\cos t}{\sin t} + \dfrac{\sin t}{1+\cos t}$

$$= \frac{\cos t}{\sin t} \cdot \frac{1+\cos t}{1+\cos t} + \frac{\sin t}{1+\cos t} \cdot \frac{\sin t}{\sin t}$$

$$= \frac{\cos t + \cos^2 t}{\sin t(1+\cos t)} + \frac{\sin^2 t}{\sin t(1+\cos t)}$$

$$= \frac{\cos t + \cos^2 t + \sin^2 t}{\sin t(1+\cos t)}$$

$$= \frac{\cos t + 1}{\sin t(1+\cos t)}$$

$$= \frac{1}{\sin t}$$

$$= \csc t$$

32. $\dfrac{\sin x}{\cos x + 1} + \dfrac{\cos x - 1}{\sin x}$

$$= \frac{\sin x}{\cos x + 1} \cdot \frac{\cos x - 1}{\cos x - 1} + \frac{\cos x - 1}{\sin x}$$

$$= \frac{\sin x(\cos x - 1)}{\cos^2 x - 1} + \frac{\cos x - 1}{\sin x}$$

$$= \frac{\sin x(\cos x - 1)}{-\sin^2 x} + \frac{\cos x - 1}{\sin x}$$

$$= \frac{\sin x(1 - \cos x)}{\sin^2 x} + \frac{\cos x - 1}{\sin x}$$

$$= \frac{1 - \cos x}{\sin x} + \frac{\cos x - 1}{\sin x}$$

$$= \frac{0}{\sin x}$$

$$= 0$$

$$= \csc t$$

30. $$1-\frac{\cos^2 x}{1+\sin x} = 1-\frac{\cos^2}{1+\sin x}\cdot\frac{1-\sin x}{1-\sin x}$$
$$= 1-\frac{\cos^2 x(1-\sin x)}{1-\sin^2 x}$$
$$= 1-\frac{\cos^2 x(1-\sin x)}{\cos^2 x}$$
$$= 1-1+\sin x$$
$$= \sin x$$

34. $$\csc^2 x\sec x = (1+\cot^2 x)\sec x$$
$$= \sec x+\cot^2 x\sec x$$
$$= \sec x+\frac{\cos^2 x}{\sin^2 x}\cdot\frac{1}{\cos x}$$
$$= \sec x+\frac{\cos x}{\sin^2 x}$$
$$= \sec x+\frac{1}{\sin x}\cdot\frac{\cos x}{\sin x}$$
$$= \sec x+\csc x\cot x$$

36. $$\frac{\csc x-\sec x}{\csc x+\sec x} = \frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}$$
$$= \frac{\frac{1}{\sin x}-\frac{1}{\cos x}}{\frac{1}{\sin x}+\frac{1}{\cos x}}\cdot\frac{\cos x}{\cos x}$$
$$= \frac{\frac{\cos x}{\sin x}-1}{\frac{\cos x}{\sin x}+1}$$
$$= \frac{\cot x-1}{\cot x+1}$$

38. $$\frac{\tan^2 x-\cot^2 x}{\tan x+\cot x} = \frac{(\tan x-\cot x)(\tan x+\cot x)}{\tan x+\cot x}$$
$$= \tan x-\cot x$$

40. $$\cot^2 2x+\cos^2 2x+\sin^2 2x = \cot^2 2x+1$$
$$= \csc^2 2x$$