5.1B HW Answers

Monday, November 20, 2017 12:17 PM

42.
$$\frac{\tan 2\theta+\cot 2\theta}{\sec 2\theta}=\frac{\frac{\sin 2\theta}{\cos 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{\frac{\sin 2\theta}{\cos 2\theta}\cdot\frac{\sin 2\theta}{\sin 2\theta}+\frac{\cos 2\theta}{\sin 2\theta}\cdot\frac{\cos 2\theta}{\cos 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{\frac{\sin^2 2\theta}{\cos 2\theta\sin 2\theta}+\frac{\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{\frac{\sin^2 2\theta+\cos^2 2\theta}{\cos 2\theta\sin 2\theta}}{\frac{1}{\cos 2\theta}}$$
$$=\frac{1}{\cos 2\theta\sin 2\theta}\div\frac{1}{\cos 2\theta}$$
$$=\frac{1}{\cos 2\theta\sin 2\theta}\cdot\frac{\cos 2\theta}{1}$$
$$=\frac{1}{\sin 2\theta}$$
$$=\csc 2\theta$$

44.
$$\frac{\cot x+\cot y}{1-\cot x\cot y}=\frac{\frac{\cos x}{\sin x}+\frac{\cos y}{\sin y}}{1-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}}$$
$$=\frac{\frac{\cos x}{\sin x}+\frac{\cos y}{\sin y}}{1-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}}\cdot\frac{\sin x\sin y}{\sin x\sin y}$$
$$=\frac{\frac{\cos x}{\sin x}\cdot\frac{\sin x\sin y}{1}+\frac{\cos y}{\sin y}\cdot\frac{\sin x\sin y}{1}}{\sin x\sin y-\frac{\cos x}{\sin x}\cdot\frac{\cos y}{\sin y}\cdot\frac{\sin x\sin y}{1}}$$
$$=\frac{\cos x\sin y+\sin x\cos y}{\sin x\sin y-\cos x\cos y}$$

46. Left side: $(\csc x-\cot x)^2=\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)^2=\left(\frac{1-\cos x}{\sin x}\right)^2=\frac{(1-\cos x)^2}{\sin^2 x}$

Right side: $\frac{1-\cos x}{1+\cos x}=\frac{1-\cos x}{1+\cos x}\cdot\frac{1-\cos x}{1-\cos x}=\frac{(1-\cos x)^2}{1-\cos^2 x}=\frac{(1-\cos x)^2}{\sin^2 x}$

[redacted]

48.
$$\frac{\cot t}{\csc t+1}=\frac{\cot t}{\csc t+1}\cdot\frac{\csc t-1}{\csc t-1}$$
$$=\frac{\cot t(\csc t-1)}{\csc^2 t-1}$$
$$=\frac{\cot t(\csc t-1)}{\cot^2 t}$$
$$=\frac{\csc t-1}{\cot t}$$

49. Left side:
$$\frac{1+\cos t}{1-\cos t}=\frac{1+\cos t}{1-\cos t}\cdot\frac{1+\cos t}{1+\cos t}$$
$$=\frac{(1+\cos t)^2}{1-\cos^2 t}$$
$$=\frac{(1+\cos t)^2}{\sin^2 t}$$

[redacted]

$$(\csc t+\cot t)^2=\left(\frac{1}{\sin t}+\frac{\cos t}{\sin t}\right)^2$$
$$=\left(\frac{1+\cos t}{\sin t}\right)^2$$
$$=\frac{(1+\cos t)^2}{\sin^2 t}$$

51.
$$\cos^4 t-\sin^4 t=\left(\cos^2 t-\sin^2 t\right)\left(\cos^2 t+\sin^2 t\right)$$
$$=\left(\cos^2 t-\sin^2 t\right)\cdot 1$$
$$=1-\sin^2 t-\sin^2 t$$
$$=1-2\sin^2 t$$

53.
$$\frac{\sin\theta-\cos\theta}{\sin\theta}+\frac{\cos\theta-\sin\theta}{\cos\theta}$$
$$=\frac{(\sin\theta-\cos\theta)\cos\theta}{\cos\theta\sin\theta}+\frac{(\cos\theta-\sin\theta)\sin\theta}{\cos\theta\sin\theta}$$
$$=\frac{\sin\theta\cos\theta-\cos^2\theta+\sin\theta\cos\theta-\sin^2\theta}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta-\left(\cos^2\theta+\sin^2\theta\right)}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta-1}{\sin\theta\cos\theta}$$
$$=\frac{2\sin\theta\cos\theta}{\sin\theta\cos\theta}-\frac{1}{\sin\theta\cos\theta}$$
$$=2-\frac{1}{\sin\theta}\cdot\frac{1}{\cos\theta}$$
$$=2-\csc\theta\sec\theta$$
$$=2-\sec\theta\csc\theta$$

56. $(\cot^2\theta+1)(\sin^2+1)$

$$=\cot^2\theta\sin^2\theta+\cot^2\theta+\sin^2\theta+1$$

$$=\frac{\cos^2\theta}{\sin^2\theta}\sin^2\theta+\cot^2\theta+\sin^2\theta+1$$

$$=\cos^2\theta+\sin^2\theta+\cot^2\theta+1$$

$$=1+\cot^2\theta+1$$

$$=\cot^2\theta+2$$

59. $\dfrac{\cos^2 x-\sin^2 x}{1-\tan^2 x}$

$$=\frac{\cos^2 x-\sin^2 x}{1-\dfrac{\sin^2 x}{\cos^2 x}}=\frac{\cos^2 x-\sin^2 x}{\dfrac{\cos^2 x-\sin^2 x}{\cos^2 x}}$$

$$=\frac{\cos^2 x-\sin^2 x}{1}\div\frac{\cos^2 x-\sin^2 x}{\cos^2 x}$$

$$=\frac{\cos^2 x-\sin^2 x}{1}\cdot\frac{\cos^2 x}{\cos^2 x-\sin^2 x}=\cos^2 x$$

60. $\dfrac{\sin x+\cos x}{\sin x}-\dfrac{\cos x-\sin x}{\cos x}$

$$=\frac{(\sin x+\cos x)\cos x}{\sin x\cos x}-\frac{(\cos x-\sin x)\sin x}{\sin x\cos x}$$

$$=\frac{\sin x\cos x+\cos^2 x-\cos x\sin x+\sin^2 x}{\sin x\cos x}$$

$$=\frac{\cos^2 x+\sin^2 x}{\sin x\cos x}$$

$$=\frac{1}{\sin x\cos x}$$

$$=\frac{1}{\sin x}\cdot\frac{1}{\cos x}$$

$$=\csc x\sec x$$

$$=\sec x\csc x$$