5.2 HW Answers

Wednesday, November 29, 2017 7:21 AM

2. $\cos(120^{\circ} - 45^{\circ})$

$= \cos 120^{\circ} \cos 45^{\circ} + \sin 120^{\circ} \sin 45^{\circ}$

$= -\frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{-\sqrt{2} + \sqrt{6}}{4}$

4. $\cos\left(\frac{2\pi}{3} - \frac{\pi}{6}\right) = \cos\frac{2\pi}{3}\cos\frac{\pi}{6} + \sin\frac{2\pi}{3}\sin\frac{\pi}{6}$

$= -\frac{1}{2} \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{1}{2}$

$= \frac{-\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$= 0$

6. **a.** $\cos 50^{\circ} \cos 5^{\circ} + \sin 50^{\circ} \sin 5^{\circ}$

$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$

Thus, $\alpha = 50^{\circ}$ and $\beta = 5^{\circ}$

b. $\cos 50^{\circ} \cos 5^{\circ} + \sin 50^{\circ} \sin 5^{\circ}$

$= \cos(50^{\circ} - 5^{\circ})$

$= \cos 45^{\circ}$

c. $\cos 45^{\circ} = \frac{\sqrt{2}}{2}$

8. **a.** $\cos\frac{5\pi}{18}\cos\frac{\pi}{9} + \sin\frac{5\pi}{18}\sin\frac{\pi}{9}$

$= \cos\alpha \cos\beta + \sin\alpha \sin\beta$

$\alpha = \frac{5\pi}{18}$ and $\beta = \frac{\pi}{9}$

b. $\cos\frac{5\pi}{18}\cos\frac{\pi}{9} + \sin\frac{5\pi}{18}\sin\frac{\pi}{9}$

$= \cos\left(\frac{5\pi}{18} - \frac{\pi}{9}\right)$

$= \cos\frac{3\pi}{18}$

$= \cos\frac{\pi}{6}$

c. $\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

14. $\sin(60^{\circ} - 45^{\circ})$

$= \sin 60^{\circ} \cos 45^{\circ} - \cos 60^{\circ} \sin 45^{\circ}$

$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2} - \frac{1}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

16. $\sin 75^{\circ} = \sin(30^{\circ} + 45^{\circ})$

$= \sin 30^{\circ} \cos 45^{\circ} + \cos 30^{\circ} \sin 45^{\circ}$

$= \frac{1}{2} \cdot \frac{\sqrt{2}}{2} + \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} + \sqrt{6}}{4}$

18. $\cos(240^{\circ} + 45^{\circ}) = \cos 240^{\circ} \cos 45^{\circ} - \sin 240^{\circ} \sin 45^{\circ}$

$= \cos(180^{\circ} + 60^{\circ}) \cos 45^{\circ} - \sin(180^{\circ} + 60^{\circ}) \sin 45^{\circ}$

$= (\cos 180^{\circ} \cos 60^{\circ} - \sin 180^{\circ} \sin 60^{\circ}) \cos 45^{\circ} - (\sin 180^{\circ} \cos 60^{\circ} + \cos 180^{\circ} \sin 60^{\circ}) \sin 45^{\circ}$

$= \left(-1 \cdot \frac{1}{2} - 0 \cdot \frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2} - \left(0 \cdot \frac{1}{2} + (-1)\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}$

$= \left(-\frac{1}{2}\right)\frac{\sqrt{2}}{2} + \left(\frac{\sqrt{3}}{2}\right)\frac{\sqrt{2}}{2}$

$= -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$

20. $\cos 105^{\circ} = \cos(45^{\circ} + 60^{\circ})$

$= \cos 45^{\circ} \cos 60^{\circ} - \sin 45^{\circ} \sin 60^{\circ}$

$= \frac{\sqrt{2}}{2} \cdot \frac{1}{2} - \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

22. $\tan\left(\frac{\pi}{3} + \frac{\pi}{4}\right) = \frac{\tan\frac{\pi}{3} + \tan\frac{\pi}{4}}{1 - \tan\frac{\pi}{3}\tan\frac{\pi}{4}}$

$= \frac{\sqrt{3} + 1}{1 - \sqrt{3} \cdot 1}$

24. $\tan\left(\frac{5\pi}{3} - \frac{\pi}{4}\right) = \frac{\tan\frac{5\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$= \frac{\tan\frac{5\pi}{3} - \tan\frac{\pi}{4}}{1 + \tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$

$$= \frac{\sqrt{3}+1}{1-\sqrt{3}\cdot 1}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{1+2\sqrt{3}+3}{1-3}$$

$$= \frac{4+2\sqrt{3}}{-2}$$

$$= -2-\sqrt{3}$$

$$= \frac{\tan\frac{5\pi}{3} - \tan\frac{\pi}{4}}{1+\tan\frac{5\pi}{3}\tan\frac{\pi}{4}}$$

$$= \frac{-\sqrt{3}-1}{1+\left(-\sqrt{3}\right)\cdot 1}$$

$$= \frac{-1-\sqrt{3}}{1-\sqrt{3}}$$

$$= \frac{-1-\sqrt{3}}{1-\sqrt{3}} \cdot \frac{1+\sqrt{3}}{1+\sqrt{3}}$$

$$= \frac{-1-2\sqrt{3}-3}{1-3}$$

$$= \frac{-4-2\sqrt{3}}{-2}$$

$$= 2+\sqrt{3}$$

26. $\sin 40^{\circ} \cos 20^{\circ} + \cos 40^{\circ} \sin 20^{\circ}$

$$= \sin(40^{\circ} + 20^{\circ})$$

$$= \sin 60^{\circ}$$

$$= \frac{\sqrt{3}}{2}$$

28. $\dfrac{\tan 50^{\circ} - \tan 20^{\circ}}{1+\tan 50^{\circ} \tan 20^{\circ}} = \tan(50^{\circ} - 20^{\circ})$

$$= \tan 30^{\circ}$$

$$= \frac{\sqrt{3}}{3}$$

30. $\sin\frac{7\pi}{12}\cos\frac{\pi}{12} - \cos\frac{7\pi}{12}\sin\frac{\pi}{12} = \sin\left(\frac{7\pi}{12} - \frac{\pi}{12}\right)$

$$= \sin\frac{6\pi}{12}$$

$$= \sin\frac{\pi}{2}$$

$$= 1$$