

## 5.2A HW Answers

Wednesday, November 29, 2017 2:35 PM

$$\begin{aligned} 9. \quad \frac{\cos(\alpha-\beta)}{\cos\alpha\sin\beta} &= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\sin\beta} \\ &= \frac{\cos\alpha}{\cos\alpha}\cdot\frac{\cos\beta}{\sin\beta} - \frac{\sin\alpha}{\cos\alpha}\cdot\frac{\sin\beta}{\sin\beta} \\ &= 1 \cdot \cot\beta + \tan\alpha \cdot 1 \\ &= \tan\alpha + \cot\beta \end{aligned}$$

$$\begin{aligned} 11. \quad \cos\left(x - \frac{\pi}{4}\right) &= \cos x \cos \frac{\pi}{4} + \sin x \sin \frac{\pi}{4} \\ &= \cos x \cdot \frac{\sqrt{2}}{2} + \sin x \cdot \frac{\sqrt{2}}{2} \\ &= \frac{\sqrt{2}}{2}(\cos x + \sin x) \end{aligned}$$

$$\begin{aligned} 31. \quad \frac{\tan \frac{\pi}{5} - \tan \frac{\pi}{30}}{1 + \tan \frac{\pi}{5} \tan \frac{\pi}{30}} &= \tan\left(\frac{\pi}{5} - \frac{\pi}{30}\right) \\ &= \tan\left(\frac{5\pi}{30}\right) = \tan\left(\frac{\pi}{6}\right) \\ &= \frac{\sqrt{3}}{3} \end{aligned}$$

$$\begin{aligned} 38. \quad \tan(\pi - x) &= \frac{\tan\pi - \tan x}{1 + \tan\pi \tan x} \\ &= \frac{0 - \tan x}{1 + 0 \cdot \tan x} \\ &= -\tan x \end{aligned}$$

$$\begin{aligned} 40. \quad \cos(\alpha+\beta) + \cos(\alpha-\beta) &= \cos\alpha\cos\beta - \sin\alpha\sin\beta \\ &+ \cos\alpha\cos\beta + \sin\alpha\sin\beta \\ &= 2\cos\alpha\cos\beta \end{aligned}$$

$$\begin{aligned} 42. \quad \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} &= \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta} \\ &= \frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta} \\ &= \tan\alpha + \tan\beta \end{aligned}$$

$$\begin{aligned} 57. \quad \sin\alpha &= \frac{3}{5} = \frac{y}{r} \\ x^2 + y^2 &= r^2 \\ x^2 + 3^2 &= 5^2 \\ x^2 + 9 &= 25 \\ x^2 &= 16 \end{aligned}$$

Because  $\alpha$  lies in quadrant I,  $x$  is positive.  
 $x = 4$

$$\begin{aligned} 32. \quad \frac{\tan \frac{\pi}{5} + \tan \frac{4\pi}{5}}{1 - \tan \frac{\pi}{5} \tan \frac{4\pi}{5}} &= \tan\left(\frac{\pi}{5} + \frac{4\pi}{5}\right) \\ &= \tan \frac{5\pi}{5} \\ &= \tan\pi \\ &= 0 \end{aligned}$$

$$\begin{aligned} 34. \quad \sin\left(x + \frac{3\pi}{2}\right) &= \sin x \cos \frac{3\pi}{2} + \cos x \sin \frac{3\pi}{2} \\ &= \sin x \cdot 0 + \cos x \cdot (-1) \\ &= -\cos x \end{aligned}$$

$$\begin{aligned} 36. \quad \cos(\pi - x) &= \cos\pi \cos x + \sin\pi \sin x \\ &= -1 \cdot \cos x + 0 \cdot \sin x \\ &= -\cos x \end{aligned}$$

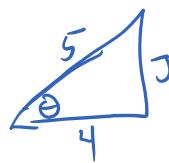
$$\begin{aligned} 46. \quad \sin(a+\beta)\sin(a-\beta) &= (\sin\alpha\cos\beta + \cos\alpha\sin\beta) \\ &\cdot (\sin\alpha\cos\beta - \cos\alpha\sin\beta) \\ &= \sin^2\alpha\cos^2\beta - \cos^2\alpha\sin^2\beta \\ &= (1 - \cos^2\alpha)\cos^2\beta \\ &- \cos^2\alpha(1 - \cos^2\beta) \\ &= \cos^2\beta - \cos^2\alpha\cos^2\beta \\ &- \cos^2\alpha + \cos^2\alpha\cos^2\beta \\ &= \cos^2\beta - \cos^2\alpha \end{aligned}$$

$$\begin{aligned} 48. \quad \frac{\cos(\alpha+\beta)}{\cos(\alpha-\beta)} &= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} \\ &= \frac{\cos\alpha\cos\beta - \sin\alpha\sin\beta}{\cos\alpha\cos\beta + \sin\alpha\sin\beta} \cdot \frac{\frac{1}{\cos\alpha\cos\beta}}{\frac{1}{\cos\alpha\cos\beta}} \\ &= \frac{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}} \\ &= \frac{1 - \tan\alpha\tan\beta}{1 + \tan\alpha\tan\beta} \end{aligned}$$

$$\begin{aligned} 51. \quad \sin 2\alpha &= \sin(\alpha + \alpha) \\ &= \sin\alpha\cos\alpha + \cos\alpha\sin\alpha \\ &= 2\sin\alpha\cos\alpha \end{aligned}$$

Because  $\beta$  lies in quadrant II,  $x$  is negative.  
 $x = -12$

$$\begin{aligned} \text{Thus, } \cos\beta &= \frac{x}{r} = \frac{-12}{13} = -\frac{12}{13}, \text{ and} \\ \tan\beta &= \frac{\sin\beta}{\cos\beta} = \frac{\frac{5}{13}}{-\frac{12}{13}} = -\frac{5}{12}. \end{aligned}$$



a.  $\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$$x^2 = 16$$

Because  $\alpha$  lies in quadrant I,  $x$  is positive.  
 $x = 4$

Thus,  $\cos \alpha = \frac{x}{r} = \frac{4}{5}$ , and

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{\frac{3}{5}}{\frac{4}{5}} = \frac{3}{4}.$$

$$\sin \beta = \frac{5}{13} = \frac{y}{r}$$

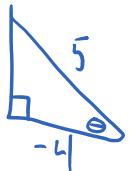
$$x^2 + y^2 = r^2$$

$$x^2 + 5^2 = 13^2$$

$$x^2 + 25 = 169$$

$$x^2 = 144$$

59.  $\tan \alpha = -\frac{3}{4} = \frac{3}{-4} = \frac{y}{x}$



$$x^2 + y^2 = r^2$$

$$(-4)^2 + 3^2 = r^2$$

$$16 + 9 = r^2$$

$$25 = r^2$$

Because  $r$  is a distance, it is positive.

$$r = 5$$

Thus,  $\cos \alpha = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$ , and

$$\sin \alpha = \frac{y}{r} = \frac{3}{5}.$$

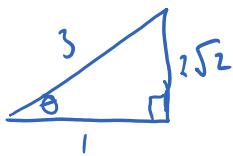
$$\cos \beta = \frac{1}{3} = \frac{x}{r}$$

$$x^2 + y^2 = r^2$$

$$1^2 + y^2 = 3^2$$

$$1 + y^2 = 9$$

$$y^2 = 8$$



61.  $\cos \alpha = \frac{8}{17} = \frac{x}{r}$

$$x^2 + y^2 = r^2$$

$$8^2 + y^2 = 17^2$$

$$64 + y^2 = 289$$

$$y^2 = 225$$

Because  $\alpha$  lies in quadrant IV,  $y$  is negative.  
 $y = -15$

Thus,  $\sin \alpha = \frac{y}{r} = \frac{-15}{17} = -\frac{15}{17}$ , and

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\frac{15}{17}}{\frac{8}{17}} = -\frac{15}{8}.$$

$$\sin \beta = -\frac{1}{r} = -\frac{1}{17} = -\frac{y}{r}$$

$$\cos \beta = -\frac{12}{13} = -\frac{12}{13}$$

$$12$$

a.  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$= \frac{4}{5} \cdot \left(-\frac{12}{13}\right) - \frac{3}{5} \cdot \frac{5}{13} = -\frac{63}{65}$$

b.  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$= \frac{3}{5} \cdot \left(-\frac{12}{13}\right) + \frac{4}{5} \cdot \frac{5}{13} = -\frac{16}{65}$$

c.  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$= \frac{\frac{3}{4} + \left(-\frac{5}{12}\right)}{1 - \frac{3}{4} \cdot \left(-\frac{5}{12}\right)} = \frac{\frac{4}{12}}{\frac{63}{48}} = \frac{16}{63}$$

Because  $\beta$  lies in quadrant I,  $y$  is positive.

$$y = \sqrt{8} = 2\sqrt{2}$$

Thus,  $\sin \beta = \frac{y}{r} = \frac{2\sqrt{2}}{3}$ , and

$$\tan \beta = \frac{\sin \beta}{\cos \beta} = \frac{\frac{2\sqrt{2}}{3}}{\frac{1}{3}} = 2\sqrt{2}.$$

a.  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$= \left(-\frac{4}{5}\right) \cdot \frac{1}{3} - \frac{3}{5} \cdot \frac{2\sqrt{2}}{3} \\ = -\frac{4}{15} - \frac{6\sqrt{2}}{15} \\ = \frac{-4 - 6\sqrt{2}}{15} \\ = -\frac{4 + 6\sqrt{2}}{15}$$

b.  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$= \frac{3}{5} \cdot \frac{1}{3} + \left(-\frac{4}{5}\right) \cdot \frac{2\sqrt{2}}{3} \\ = \frac{3}{15} - \frac{8\sqrt{2}}{15} \\ = \frac{3 - 8\sqrt{2}}{15}$$

c.  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$= \frac{-\frac{3}{4} + 2\sqrt{2}}{1 - \left(-\frac{3}{4}\right)(2\sqrt{2})} \\ = \frac{\frac{-3 + 8\sqrt{2}}{4}}{\frac{4 + 6\sqrt{2}}{4}} \\ = \frac{-3 + 8\sqrt{2}}{4 + 6\sqrt{2}} \cdot \frac{(4 - 6\sqrt{2})}{(4 - 6\sqrt{2})} \\ = \frac{-108 + 50\sqrt{2}}{-56} \\ = \frac{54 - 25\sqrt{2}}{28}$$

Because  $\beta$  lies in quadrant III,  $x$  is negative.

$$x = -\sqrt{3}$$

Thus,  $\cos \beta = \frac{x}{r} = \frac{-\sqrt{3}}{2} = -\frac{\sqrt{3}}{2}$ , and

$$\tan \beta = \frac{\sin \beta}{\cos \beta} = \frac{-\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}.$$

b.  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$= \left(-\frac{15}{17}\right) \cdot \left(-\frac{\sqrt{3}}{2}\right) + \frac{8}{17} \cdot \left(-\frac{1}{2}\right) \\ = \frac{15\sqrt{3} - 8}{34}$$

c.  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$= \frac{-\frac{15}{8} + \frac{\sqrt{3}}{3}}{1 - \left(-\frac{15}{8}\right)\left(\frac{\sqrt{3}}{3}\right)} \\ = \frac{-45 + 8\sqrt{3}}{-42}$$

$$\tan \alpha = \frac{\sin \alpha}{\cos \alpha} = \frac{-\frac{17}{8}}{\frac{8}{17}} = -\frac{17}{8}$$

$$\sin \beta = -\frac{1}{2} = \frac{-1}{2} = \frac{y}{r}$$

$$x^2 + y^2 = r^2$$

$$x^2 + (-1)^2 = 2^2$$

$$x^2 + 1 = 4$$

$$x^2 = 3$$

$$\begin{aligned} &= \frac{v}{17} \cdot \left( -\frac{v}{2} \right) - \left( -\frac{u}{17} \right) \cdot \left( -\frac{u}{2} \right) \\ &= \frac{-8\sqrt{3} - 15}{34} \\ &= -\frac{8\sqrt{3} + 15}{34} \end{aligned}$$

$$\begin{aligned} &= 1 - \left( -\frac{15}{8} \right) \left( \frac{\sqrt{3}}{3} \right) \\ &= \frac{-45 + 8\sqrt{3}}{24} \\ &= \frac{24}{24 + 15\sqrt{3}} \\ &= \frac{-45 + 8\sqrt{3}}{24 + 15\sqrt{3}} \cdot \frac{24 - 15\sqrt{3}}{24 - 15\sqrt{3}} \\ &= \frac{-1440 + 867\sqrt{3}}{-99} \\ &= \frac{489 - 289\sqrt{3}}{33} \end{aligned}$$

63.  $\tan \alpha = \frac{3}{4} = \frac{y}{x}$

Because  $\alpha$  lies in quadrant III,  $x$  and  $y$  are negative.

$$r^2 = x^2 + y^2$$

$$r^2 = (-4)^2 + (-3)^2$$

$$r^2 = 25$$

$$r = 5$$

$$\sin \alpha = \frac{y}{r} = \frac{-3}{5} = -\frac{3}{5}$$

$$\cos \alpha = \frac{x}{r} = \frac{-4}{5} = -\frac{4}{5}$$

$$\cos \beta = \frac{1}{4} = \frac{x}{r}$$

Because  $\beta$  lies in quadrant IV,  $y$  is negative.

$$x^2 + y^2 = r^2$$

$$1^2 + y^2 = 4^2$$

$$y^2 = 15$$

$$y = -\sqrt{15}$$

$$\sin \beta = \frac{y}{r} = \frac{-\sqrt{15}}{4} = -\frac{\sqrt{15}}{4}$$

$$\tan \beta = \frac{y}{x} = \frac{-\sqrt{15}}{1} = -\sqrt{15}$$

a.  $\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$

$$\begin{aligned} &= -\frac{4}{5} \left( \frac{1}{4} \right) - \left( -\frac{3}{5} \right) \left( -\frac{\sqrt{15}}{4} \right) \\ &= -\frac{4}{20} - \frac{3\sqrt{15}}{20} \\ &= -\frac{4+3\sqrt{15}}{20} \end{aligned}$$

b.  $\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$

$$\begin{aligned} &= \left( -\frac{3}{5} \right) \left( \frac{1}{4} \right) + \left( -\frac{4}{5} \right) \left( -\frac{\sqrt{15}}{4} \right) \\ &= -\frac{3}{20} + \frac{4\sqrt{15}}{20} \\ &= -\frac{-3+4\sqrt{15}}{20} \end{aligned}$$

c.  $\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$

$$\begin{aligned} &= \frac{\frac{3}{4} + (-\sqrt{15})}{1 - \frac{3}{4}(-\sqrt{15})} \\ &= \frac{\frac{3}{4} - \frac{4\sqrt{15}}{4}}{\frac{4}{4} + \frac{3\sqrt{15}}{4}} \end{aligned}$$

$$\begin{aligned} &= \frac{3-4\sqrt{15}}{4+3\sqrt{15}} \\ &= \frac{3-4\sqrt{15}}{4+3\sqrt{15}} \cdot \frac{4-3\sqrt{15}}{4-3\sqrt{15}} \\ &= \frac{12-9\sqrt{15}-16\sqrt{15}+180}{16-135} \\ &= \frac{192-25\sqrt{15}}{-119} \\ &= \frac{-192+25\sqrt{15}}{119} \end{aligned}$$