5.3 HW Answers

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1. $\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$

3. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(\frac{3}{4}\right)}{1-\left(\frac{3}{4}\right)^2} = \frac{\frac{3}{2}}{1-\frac{9}{16}}$

$= \frac{\frac{3}{2}}{\frac{7}{16}} = \left(\frac{3}{2}\right)\left(\frac{16}{7}\right) = \frac{24}{7}$

Use this information to solve problems 4, 5, and 6.

$\tan\alpha = \frac{7}{24} = \frac{y}{x}$

Because r is a distance it is positive.

$x^2 + y^2 = r^2$

$24^2 + 7^2 = r^2$

$576 + 49 = r^2$

$625 = r^2$

$r = 25$

$\sin\alpha = \frac{y}{r} = \frac{7}{25}$

$\cos\alpha = \frac{x}{r} = \frac{24}{25}$

5. $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$= \left(\frac{24}{25}\right)^2 - \left(\frac{7}{25}\right)^2 = \frac{576}{625} - \frac{49}{625}$

$= \frac{527}{625}$

7. $\sin\theta = \frac{15}{17} = \frac{y}{r}$

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$= 2\left(\frac{15}{17}\right)\left(-\frac{8}{17}\right) = -\frac{240}{289}$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$= \left(-\frac{8}{17}\right)^2 - \left(\frac{15}{17}\right)^2 = \frac{64}{289} - \frac{225}{289}$

$= -\frac{161}{289}$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$= \frac{2\left(-\frac{15}{8}\right)}{1-\left(-\frac{15}{8}\right)^2} = \frac{-\frac{15}{4}}{1-\frac{225}{64}} = \frac{-\frac{15}{4}}{-\frac{161}{64}}$

$= \left(-\frac{15}{4}\right)\left(-\frac{64}{161}\right) = \frac{240}{161}$

9. $\cos\theta = \frac{24}{25} = \frac{x}{r}$

Because θ lies in quadrant IV, y is negative.

$$x^2 + y^2 = r^2$$
$$24^2 + y^2 = 25^2$$
$$y^2 = 25^2 - 24^2 = 49$$
$$y = -\sqrt{49} = -7$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(-\frac{7}{25}\right)\left(\frac{24}{25}\right) = -\frac{336}{625}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(\frac{24}{25}\right)^2 - \left(-\frac{7}{25}\right)^2$$
$$= \frac{576}{625} - \frac{49}{625} = \frac{527}{625}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\left(-\frac{7}{24}\right)}{1-\left(-\frac{7}{24}\right)^2} = \frac{-\frac{7}{12}}{1-\frac{49}{576}} = \frac{-\frac{7}{12}}{\frac{527}{576}}$$
$$= \left(-\frac{7}{12}\right)\left(\frac{576}{527}\right) = -\frac{336}{527}$$

11. $\cot\theta = 2 = \frac{-2}{-1} = \frac{x}{y}$

Because r is a distance, it is positive.

$$r^2 = x^2 + y^2$$
$$r^2 = (-2)^2 + (-1)^2$$
$$r^2 = 5$$
$$r = \sqrt{5}$$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(-\frac{1}{\sqrt{5}}\right)\left(-\frac{2}{\sqrt{5}}\right) = \frac{4}{5}$$

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(-\frac{2}{\sqrt{5}}\right)^2 - \left(-\frac{1}{\sqrt{5}}\right)^2$$
$$= \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\left(\frac{1}{2}\right)}{1-\left(\frac{1}{2}\right)^2} = \frac{1}{1-\frac{1}{4}} = \frac{1}{\frac{3}{4}}$$
$$= (1)\left(\frac{4}{3}\right) = \frac{4}{3}$$

13. $\sin\theta = -\frac{9}{41} = \frac{-9}{41} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$$x^2 + y^2 = r^2$$
$$x^2 + (-9)^2 = 41^2$$
$$x^2 = 1600$$
$$x = -\sqrt{1600}$$
$$x = -40$$

Now we use values for x, y, and r to find the required values.

b. $\cos 2\theta = \cos^2\theta - \sin^2\theta$

$$= \left(-\frac{40}{41}\right)^2 - \left(-\frac{9}{41}\right)^2$$
$$= \frac{1600}{1681} - \frac{81}{1681}$$
$$= \frac{1519}{1681}$$

c. $\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta}$

$2\left(-\frac{9}{\ }\right)$ 9 9

$x = -40$

Now we use values for x, y, and r to find the required values.

a. $\sin 2\theta = 2\sin\theta\cos\theta$

$$= 2\left(-\frac{9}{41}\right)\left(-\frac{40}{41}\right) = \frac{720}{1681}$$

c. $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

$$= \frac{2\left(\frac{9}{40}\right)}{1-\left(\frac{9}{40}\right)^2} = \frac{\frac{9}{20}}{1-\frac{81}{1600}} = \frac{\frac{9}{20}}{\frac{1519}{1600}}$$

$$= \left(\frac{9}{20}\right)\left(\frac{1600}{1519}\right) = \frac{720}{1519}$$

15. The given expression is the right side of the formula for $\sin 2\theta$ with $\theta = 15°$.

$2\sin 15°\cos 15° = \sin(2\cdot 15°)$

$$= \sin 30° = \frac{1}{2}$$

17. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = 75°$.

$\cos^2 75° - \sin^2 75° = \cos(2\cdot 75°)$

$$= \cos 150° = -\frac{\sqrt{3}}{2}$$

19. The given expression is the right side of the formula for $\cos 2\theta$ with $\theta = \dfrac{\pi}{8}$.

$$2\cos^2\frac{\pi}{8} - 1 = \cos\left(2\cdot\frac{\pi}{8}\right)$$

$$= \cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}$$

21. The given expression is the right side of the formula for $\tan 2\theta$ with $\theta = \dfrac{\pi}{12}$.

$$\frac{2\tan\frac{\pi}{12}}{1-\tan^2\frac{\pi}{12}} = \tan\left(2\cdot\frac{\pi}{12}\right) = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$$

23. $\dfrac{2\tan\theta}{1+\tan^2\theta} = \dfrac{2\cdot\frac{\sin\theta}{\cos\theta}}{\frac{\cos^2\theta}{\cos^2\theta}+\frac{\sin^2\theta}{\cos^2\theta}}$

$$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{\frac{2\sin\theta}{\cos\theta}}{\frac{1}{\cos^2\theta}}$$

$$= \frac{2\sin\theta}{\cos\theta}\cdot\frac{\cos^2\theta}{1}$$

$$= 2\sin\theta\cos\theta$$

$$= \sin 2\theta$$

25. $$\begin{aligned}(\sin\theta+\cos\theta)^2 &= \sin^2\theta+2\sin\theta\cos\theta+\cos^2\theta\\ &= \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta\\ &= 1+2\sin\theta\cos\theta\\ &= 1+\sin 2\theta\end{aligned}$$

27. $$\begin{aligned}\sin^2 x+\cos 2x &= \sin^2 x+\cos^2 x-\sin^2 x\\ &= \cos^2 x\end{aligned}$$

28. $$\begin{aligned}\frac{\cos 2x}{\cos^2 x} &= \frac{1-2\sin^2 x}{\cos^2 x}\\ &= \frac{1-\sin^2 x-\sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x-\sin^2 x}{\cos^2 x}\\ &= \frac{\cos^2 x}{\cos^2 x}-\frac{\sin^2 x}{\cos^2 x}\\ &= 1-\tan^2 x\end{aligned}$$

29. $$\begin{aligned}\frac{\sin 2x}{1-\cos 2x} &= \frac{2\sin x\cos x}{1-\left(\cos^2 x-\sin^2 x\right)}\\ &= \frac{2\sin x\cos x}{1-\cos^2 x+\sin^2 x}\\ &= \frac{2\sin x\cos x}{\sin^2+\sin^2 x}\\ &= \frac{2\sin x\cos x}{2\sin^2 x}\\ &= \frac{\cos x}{\sin x}\\ &= \cot x\end{aligned}$$

30. $$\begin{aligned}\frac{1+\cos 2x}{\sin 2x} &= \frac{1+\cos^2 x-\sin^2 x}{2\sin x\cos x}\\ &= \frac{1-\sin^2 x+\cos^2 x}{2\sin x\cos x}\\ &= \frac{\cos^2 x+\cos^2 x}{2\sin x\cos x}\\ &= \frac{2\cos^2 x}{2\sin x\cos x}\\ &= \frac{\cos x}{\sin x}\\ &= \cot x\end{aligned}$$

31. $$\begin{aligned}\tan t\cos 2t &= \frac{\sin t}{\cos t}\cdot\left(2\cos^2 t-1\right)\\ &= \frac{2\sin t\cos^2 t}{\cos t}-\frac{\sin t}{\cos t}\\ &= 2\sin t\cos t-\tan t\\ &= \sin 2t-\tan t\end{aligned}$$