

5.3B HW Answers

Monday, December 4, 2017 1:00 PM

13. $\sin \theta = -\frac{9}{41} = \frac{-9}{41} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$$x^2 + y^2 = r^2$$

$$x^2 + (-9)^2 = 41^2$$

$$x^2 = 1600$$

$$x = -\sqrt{1600}$$

$$x = -40$$

Now we use values for x , y , and r to find the required values.

(13) a. $\sin 2\theta = 2\sin \theta \cos \theta$

$$= 2\left(-\frac{9}{41}\right)\left(-\frac{40}{41}\right) = \frac{720}{1681}$$

b. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$$\begin{aligned} &= \left(-\frac{40}{41}\right)^2 - \left(-\frac{9}{41}\right)^2 \\ &= \frac{1600}{1681} - \frac{81}{1681} \\ &= \frac{1519}{1681} \end{aligned}$$

c. $\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$

$$\begin{aligned} &= \frac{2\left(\frac{9}{40}\right)}{1 - \left(\frac{9}{40}\right)^2} = \frac{\frac{9}{20}}{1 - \frac{81}{1600}} = \frac{\frac{9}{20}}{\frac{1519}{1600}} \\ &= \left(\frac{9}{20}\right)\left(\frac{1600}{1519}\right) = \frac{720}{1519} \end{aligned}$$

55. $\tan \alpha = \frac{4}{3} = \frac{-4}{-3} = \frac{y}{x}$

Because r is a distance, it is positive.

$$r^2 = x^2 + y^2$$

$$r^2 = (-4)^2 + (-3)^2$$

$$r^2 = 25$$

$$r = 5$$

Since $180^\circ < \alpha < 270^\circ$, then $90^\circ < \frac{\alpha}{2} < 135^\circ$.

Therefore $\frac{\alpha}{2}$ lies in quadrant II.

Thus, $\sin \frac{\alpha}{2} > 0$, $\cos \frac{\alpha}{2} < 0$, and $\tan \frac{\alpha}{2} < 0$.

55.) a. $\sin \frac{\alpha}{2} = \sqrt{\frac{1-\cos \alpha}{2}} = \sqrt{\frac{1-\left(-\frac{3}{5}\right)}{2}}$
 $= \sqrt{\frac{8}{2}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$

14. $\sin \theta = -\frac{2}{3} = \frac{-2}{3} = \frac{y}{r}$

Because θ lies in quadrant III, x is negative.

$$x^2 + y^2 = r^2$$

$$x^2 + (-2)^2 = 3^2$$

$$x^2 = 5$$

$$x = -\sqrt{5}$$

Now we use values for x , y , and r to find the required values.

(14) a. $\sin 2\theta = 2\sin \theta \cos \theta$

$$= 2\left(-\frac{2}{3}\right)\left(-\frac{\sqrt{5}}{3}\right) = \frac{4\sqrt{5}}{9}$$

b. $\cos 2\theta = \cos^2 \theta - \sin^2 \theta$

$$\begin{aligned} &= \left(-\frac{\sqrt{5}}{3}\right)^2 - \left(-\frac{2}{3}\right)^2 \\ &= \frac{5}{9} - \frac{4}{9} = \frac{1}{9} \end{aligned}$$

c. $\tan 2\theta = \frac{2\tan \theta}{1 - \tan^2 \theta}$

$$\begin{aligned} &= \frac{2 \cdot \frac{2}{\sqrt{5}}}{1 - \left(\frac{2}{\sqrt{5}}\right)^2} = \frac{\frac{4}{\sqrt{5}}}{1 - \frac{4}{5}} \\ &= \frac{\frac{4}{\sqrt{5}}}{\frac{1}{5}} = \frac{4\sqrt{5}}{4} \cdot \frac{5}{1} = 4\sqrt{5} \end{aligned}$$

56. $\tan \alpha = \frac{8}{15} = \frac{-8}{-15} = \frac{y}{x}$

Because r is a distance, it is positive.

$$r^2 = x^2 + y^2$$

$$r^2 = (-15)^2 + (-8)^2$$

$$r^2 = 289$$

$$r = \sqrt{289}$$

$$r = 17$$

Since $180^\circ < \alpha < 270^\circ$, then $90^\circ < \frac{\alpha}{2} < 135^\circ$.

Therefore $\frac{\alpha}{2}$ lies in quadrant II.

Thus, $\sin \frac{\alpha}{2} > 0$, $\cos \frac{\alpha}{2} < 0$ and $\tan \frac{\alpha}{2} < 0$.

56.) a. $\sin \frac{\alpha}{2} = \sqrt{\frac{1-\cos \alpha}{2}} = \sqrt{\frac{1-\left(-\frac{15}{17}\right)}{2}}$
 $= \sqrt{\frac{32}{2}} = \sqrt{\frac{16}{17}} = \frac{4}{\sqrt{17}} = \frac{4\sqrt{17}}{17}$

b. $\cos \frac{\alpha}{2} = -\sqrt{1 + \left(-\frac{15}{17}\right)^2}$

$$= -\sqrt{\frac{2}{17}}$$

$$= -\sqrt{\frac{1}{17}}$$

$$= -\frac{1}{\sqrt{17}}$$

b.) $= -\frac{\sqrt{17}}{17}$

c. $\tan \frac{\alpha}{2} = \frac{1 - \cos \alpha}{\sin \alpha}$
 $= \frac{1 - \left(-\frac{15}{17}\right)}{\frac{4}{\sqrt{17}}} = \frac{1 + \frac{15}{17}}{\frac{4}{\sqrt{17}}} = \frac{\frac{32}{17}}{\frac{4}{\sqrt{17}}} = \frac{32}{17} \cdot \frac{\sqrt{17}}{4} = \frac{32\sqrt{17}}{68} = \frac{8\sqrt{17}}{17}$

$$= \sqrt{\frac{8}{5}} = \sqrt{\frac{4}{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\text{b. } \cos \frac{\alpha}{2} = -\sqrt{\frac{1+\cos \alpha}{2}} = -\sqrt{\frac{1+\left(-\frac{3}{5}\right)}{2}}$$

$$= -\sqrt{\frac{2}{5}} = -\sqrt{\frac{1}{5}} = -\frac{1}{\sqrt{5}} = -\frac{\sqrt{5}}{5}$$

$$\text{c. } \tan \frac{\alpha}{2} = \frac{1-\cos \alpha}{\sin \alpha} = \frac{1-\left(-\frac{3}{5}\right)}{-\frac{4}{5}}$$

$$= \frac{\frac{8}{5}}{-\frac{4}{5}} = \frac{8}{-4} = -2$$

57. $\sec \alpha = -\frac{13}{5} = \frac{13}{-5} = \frac{r}{x}$

Because α lies in quadrant II, y is positive.

$$x^2 + y^2 = r^2$$

$$(-5)^2 + y^2 = (13)^2$$

$$y^2 = 144$$

$$y = 12$$

Since $\frac{\pi}{2} < \alpha < \pi$, then $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$. Therefore $\frac{\alpha}{2}$ lies in quadrant I.

Thus, $\sin \frac{\alpha}{2} > 0$, $\cos \frac{\alpha}{2} > 0$, and $\tan \frac{\alpha}{2} > 0$.

57) a. $\sin \frac{\alpha}{2} = \sqrt{\frac{1-\cos \alpha}{2}} = \sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}}$

$$= \sqrt{\frac{18}{26}} = \sqrt{\frac{9}{13}} = \frac{3}{\sqrt{13}}$$

$$= \frac{3\sqrt{13}}{13}$$

b. $\cos \frac{\alpha}{2} = \sqrt{\frac{1+\cos \alpha}{2}} = \sqrt{\frac{1+\left(-\frac{5}{13}\right)}{2}}$

$$= \sqrt{\frac{8}{26}} = \sqrt{\frac{4}{13}} = \frac{2}{\sqrt{13}}$$

$$= \frac{2\sqrt{13}}{13}$$

c. $\tan \frac{\alpha}{2} = \frac{1-\cos \alpha}{\sin \alpha} = \frac{1-\left(-\frac{5}{13}\right)}{\frac{12}{13}}$

$$= \frac{13+5}{12} = \frac{18}{12} = \frac{3}{2}$$

$$= \sqrt{-\frac{1}{2}}$$

$$= \sqrt{\frac{32}{2}}$$

$$= \sqrt{\frac{16}{17}}$$

$$= \frac{4}{\sqrt{17}}$$

$$= \frac{4\sqrt{17}}{17}$$

$$= -\frac{8}{17}$$

$$= \frac{32}{8} = \frac{32}{-8} = -4$$

$$= -\frac{17}{17}$$

58. $\sec \alpha = -3 = \frac{3}{-1} = \frac{r}{x}$

Because α lies in quadrant II, y is positive.

$$x^2 + y^2 = r^2$$

$$(-1)^2 + y^2 = 3^2$$

$$y^2 = 8$$

$$y = \sqrt{8}$$

$$y = 2\sqrt{2}$$

Since $\frac{\pi}{2} < \alpha < \pi$, then $\frac{\pi}{4} < \frac{\alpha}{2} < \frac{\pi}{2}$. Therefore $\frac{\alpha}{2}$ lies in quadrant I.

Thus, $\sin \frac{\alpha}{2} > 0$, $\cos \frac{\alpha}{2} > 0$, and $\tan \frac{\alpha}{2} > 0$.

58) a. $\sin \frac{\alpha}{2} = \sqrt{\frac{1-\cos \alpha}{2}} = \sqrt{\frac{1-\left(-\frac{1}{3}\right)}{2}}$

$$= \sqrt{\frac{4}{3}}$$

$$= \frac{\sqrt{4}}{\sqrt{3}}$$

$$= \frac{2}{\sqrt{3}}$$

$$= \frac{\sqrt{2}}{\sqrt{3}}$$

$$= \frac{\sqrt{6}}{3}$$

b. $\cos \frac{\alpha}{2} = \sqrt{\frac{1+\cos \alpha}{2}}$

$$= \sqrt{\frac{1+\left(-\frac{1}{3}\right)}{2}}$$

$$= \sqrt{\frac{2}{3}}$$

$$= \sqrt{\frac{2}{3}}$$

$$= \frac{1}{\sqrt{3}}$$

b.) $= \frac{\sqrt{3}}{3}$

c. $\tan \frac{\alpha}{2} = \frac{1-\cos \alpha}{\sin \alpha}$

$$= \frac{1-\left(-\frac{1}{3}\right)}{\frac{12}{13}}$$

$$= \frac{2\sqrt{2}}{3}$$

$$= \frac{3+1}{2\sqrt{2}}$$

$$= \frac{4}{2\sqrt{2}}$$

$$= \sqrt{2}$$

$$\begin{aligned} \text{X. } \cos^2 \frac{\theta}{2} &= \frac{1 + \cos 2\left(\frac{\theta}{2}\right)}{2} \\ &= \frac{1 + \cos \theta}{2} \\ &= \frac{1 + \cos \theta}{2} \cdot \frac{\frac{\sin \theta}{\cos \theta}}{\frac{\sin \theta}{\cos \theta}} \\ &= \frac{\frac{\sin \theta + \sin \theta}{\cos \theta}}{2 \cdot \frac{\sin \theta}{\cos \theta}} \\ &= \frac{\tan \theta + \sin \theta}{2 \tan \theta} \\ &= \frac{\sin \theta + \tan \theta}{2 \tan \theta} \end{aligned}$$

#39) $\frac{\sqrt{3}}{2}$ 40) $-\frac{\sqrt{3}}{3}$

41) $\frac{\sqrt{2 - \sqrt{2}}}{2}$ 42) $2 - \sqrt{3}$

#39) $\cos^2(15^\circ) - \sin^2(15^\circ) = \cos 2(15^\circ)$
 $= \cos 30^\circ$