# 5.5C HW Answers

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**63.**

$$\begin{aligned}
2\cos^2 x + \sin x - 1 &= 0\\
2\left(1-\sin^2 x\right) + \sin x - 1 &= 0\\
2 - 2\sin^2 x + \sin x - 1 &= 0\\
-2\sin^2 x + \sin x + 1 &= 0\\
2\sin^2 x - \sin x - 1 &= 0\\
(2\sin x + 1)(\sin x - 1) &= 0
\end{aligned}$$

$2\sin x + 1 = 0$ or $\sin x - 1 = 0$

$2\sin x = -1$ $\qquad$ $\sin x = 1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$ $\quad x = \frac{11\pi}{6}$ $\qquad x = \frac{\pi}{2}$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{2}, \frac{7\pi}{6}$, and $\frac{11\pi}{6}$.

**64.**

$$\begin{aligned}
2\cos^2 x - \sin x - 1 &= 0\\
2\left(1-\sin^2 x\right) - \sin x - 1 &= 0\\
2 - 2\sin^2 x - \sin x - 1 &= 0\\
-2\sin^2 x - \sin x + 1 &= 0\\
2\sin^2 x + \sin x - 1 &= 0\\
(2\sin x - 1)(\sin x + 1) &= 0
\end{aligned}$$

$2\sin x - 1 = 0$ or $\sin x + 1 = 0$

$2\sin x = 1$ $\qquad$ $\sin x = -1$

$\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$ $\quad x = \frac{5\pi}{6}$ $\qquad x = \frac{3\pi}{2}$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

**65.**

$$\begin{aligned}
\sin^2 x - 2\cos x - 2 &= 0\\
1 - \cos^2 x - 2\cos x - 2 &= 0\\
-\cos^2 x - 2\cos x - 1 &= 0\\
\cos^2 x + 2\cos x + 1 &= 0\\
(\cos x + 1)(\cos x + 1) &= 0\\
\cos x + 1 &= 0\\
\cos x &= -1\\
x &= \pi
\end{aligned}$$

The solution in the interval $[0, 2\pi)$ is $\pi$.

**66.**

$$\begin{aligned}
4\sin^2 x + 4\cos x - 5 &= 0\\
4\left(1-\cos^2 x\right) + 4\cos x - 5 &= 0\\
4 - 4\cos^2 x + 4\cos x - 5 &= 0\\
-4\cos^2 x + 4\cos x - 1 &= 0\\
4\cos^2 x - 4\cos x + 1 &= 0\\
(2\cos x - 1)(2\cos x - 1) &= 0\\
2\cos x - 1 &= 0\\
2\cos x &= 1\\
\cos x &= \frac{1}{2}
\end{aligned}$$

$x = \frac{\pi}{3}$ $\quad x = \frac{5\pi}{3}$

The solutions in the interval $[0, 2\pi)$ are

$\frac{\pi}{3}$ and $\frac{5\pi}{3}$.

**67.**

$$\begin{aligned}
4\cos^2 x &= 5 - 4\sin x\\
4\cos^2 x + 4\sin x - 5 &= 0\\
4\left(1-\sin^2 x\right) + 4\sin x - 5 &= 0\\
4 - 4\sin^2 x + 4\sin x - 5 &= 0
\end{aligned}$$

**68.**

$$\begin{aligned}
3\cos^2 x &= \sin^2 x\\
3\left(1-\sin^2 x\right) &= \sin^2 x\\
3 - 3\sin^2 x - \sin^2 x &= 0\\
-4\sin^2 x &= -3
\end{aligned}$$

$$
\begin{aligned}
4\left(1-\sin^2 x\right)+4\sin x-5&=0\\
4-4\sin^2 x+4\sin x-5&=0\\
-4\sin^2 x+4\sin x-1&=0\\
4\sin^2 x-4\sin x+1&=0\\
(2\sin x-1)(2\sin x-1)&=0\\
2\sin x-1&=0\\
2\sin x&=1\\
\sin x&=\frac{1}{2}\\
x=\frac{\pi}{6}\quad x&=\frac{5\pi}{6}
\end{aligned}
$$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{6}$ and $\frac{5\pi}{6}$.

$$
\begin{aligned}
3-3\sin^2 x-\sin^2 x&=0\\
-4\sin^2 x&=-3\\
\sin^2 x&=\frac{3}{4}\\
\sin x&=\pm\sqrt{\frac{3}{4}}\\
\sin x&=\pm\frac{\sqrt{3}}{2}
\end{aligned}
$$

$$\sin x=\frac{\sqrt{3}}{2}\quad\text{or}\quad \sin x=-\frac{\sqrt{3}}{2}$$

$$x=\frac{\pi}{3}\quad x=\frac{2\pi}{3}\quad x=\frac{4\pi}{3}\quad x=\frac{5\pi}{3}$$

The solutions in the interval $[0,2\pi)$ are $\frac{\pi}{3},\frac{2\pi}{3},\frac{4\pi}{3}$, and $\frac{5\pi}{3}$.

**69.**

$$
\begin{aligned}
\sin 2x&=\cos x\\
2\sin x\cos x&=\cos x\\
2\sin x\cos x-\cos x&=0\\
\cos x(2\sin x-1)&=0
\end{aligned}
$$

$$\cos x=0\quad\text{or}\quad 2\sin x-1=0$$

$$2\sin x=1$$

$$\sin x=\frac{1}{2}$$

$$x=\frac{\pi}{2}\quad x=\frac{3\pi}{2}\qquad x=\frac{\pi}{6}\quad x=\frac{5\pi}{6}$$

The solutions in the interval $[0,2\pi)$ are

$\frac{\pi}{6},\frac{\pi}{2},\frac{5\pi}{6}$,and $\frac{3\pi}{2}$.

**70.**

$$
\begin{aligned}
\sin 2x&=\sin x\\
2\sin x\cos x&=\sin x\\
2\sin x\cos x-\sin x&=0\\
\sin x(2\cos x-1)&=0
\end{aligned}
$$

$$\sin x=0\quad\text{or}\quad 2\cos x-1=0$$

$$2\cos x=1$$

$$\cos x=\frac{1}{2}$$

$$x=0\quad x=\pi\quad x=\frac{\pi}{3}\quad x=\frac{5\pi}{3}$$

The solutions in the interval $[0,2\pi)$ are

$0,\frac{\pi}{3},\pi$, and $\frac{5\pi}{3}$.

**71.**

$$
\begin{aligned}
\cos 2x&=\cos x\\
2\cos^2 x-1&=\cos x\\
2\cos^2 x-1-\cos x&=0\\
2\cos^2 x-\cos x-1&=0\\
(2\cos x+1)(\cos x-1)&=0
\end{aligned}
$$

$$2\cos x+1=0\quad\text{or}\quad\cos x-1=0$$

$$2\cos x=-1\qquad\qquad \cos x=1$$

1

**72.**

$$
\begin{aligned}
\cos 2x&=\sin x\\
1-2\sin^2 x&=\sin x\\
1-2\sin^2 x-\sin x&=0\\
-2\sin^2 x-\sin x+1&=0\\
2\sin^2 x+\sin x-1&=0\\
(2\sin x-1)(\sin x+1)&=0
\end{aligned}
$$

$$2\sin x-1=0\quad\text{or}\quad\sin x+1=0$$

$$2\sin x=1\qquad\qquad \sin x=-1$$

$$2\cos x+1=0 \quad \text{or} \quad \cos x-1=0$$
$$2\cos x=-1 \qquad \cos x=1$$
$$\cos x=-\frac{1}{2} \qquad \cos x=1$$
$$x=\frac{2\pi}{3} \quad x=\frac{4\pi}{3} \qquad x=0$$

The solutions in the interval $[0,2\pi)$ are

$0, \frac{2\pi}{3}$, and $\frac{4\pi}{3}$.

$$(2\sin x-1)(\sin x+1)=0$$
$$2\sin x-1=0 \quad \text{or} \quad \sin x+1=0$$
$$2\sin x=1 \qquad \sin x=-1$$
$$\sin x=\frac{1}{2}$$
$$x=\frac{\pi}{6} \quad x=\frac{5\pi}{6} \qquad x=\frac{3\pi}{2}$$

The solutions in the interval $[0,2\pi)$ are

$\frac{\pi}{6}, \frac{5\pi}{6}$, and $\frac{3\pi}{2}$.

**73.**
$$\cos 2x+5\cos x+3=0$$
$$2\cos^2 x-1+5\cos x+3=0$$
$$2\cos^2 x+5\cos x+2=0$$
$$(2\cos x+1)(\cos x+2)=0$$
$$2\cos x+1=0 \quad \text{or} \quad \cos x+2=0$$
$$2\cos x=-1 \qquad \cos x=-2$$
$$\cos x=-\frac{1}{2}$$
$$x=\frac{2\pi}{3} \quad x=\frac{4\pi}{3} \qquad \cos x \text{ cannot be less than } -1$$

The solutions in the interval $[0,2\pi)$ are $\frac{2\pi}{3}$ and $\frac{4\pi}{3}$.

**78.**
$$\sin x+\cos x=-1$$
$$(\sin x+\cos x)^2=(-1)^2$$
$$\sin^2 x+2\sin x\cos x+\cos^2 x=1$$
$$\sin^2 x+\cos^2 x+2\sin x\cos x=1$$
$$1+2\sin x\cos x=1$$
$$2\sin x\cos x=0$$
$$\sin x\cos x=0$$
$$\sin x=0 \quad \text{or} \quad \cos x=0$$
$$x=0 \qquad x=\frac{\pi}{2}$$
$$x=\pi \qquad x=\frac{3\pi}{2}$$

After checking these proposed solutions, the actual solutions in the interval $[0,2\pi)$ are $\pi$ and $\frac{3\pi}{2}$.