Assignments for Algebra 2
Unit 5: Graphing and Writing Quadratic Functions

Alg. 2 – Unit 5 Notes – Graphing Quadratic Functions (Parabolas)

Day 1 – Graph Quadratic Functions in Standard Form

Objectives: Graph functions expressed symbolically by hand and show key features of the graph, including intercepts, vertex, maximum and minimum values, and end behaviors.

Quadratic Function - A function in standard form ax^2+bx+c where $a \neq 0$.

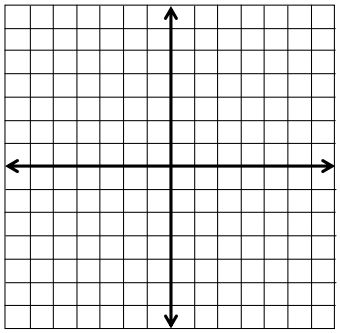
Parabola – the graph of a quadratic function.

Vertex - The lowest or highest point on on a parabola.

Axis of symmetry – divides the parabola into mirror images and passes through the vertex.

Minimum and maximum value – the vertex's y-coordinate is a maximum if a > 0 or a minimum if a < 0.

PARENT FUNCTION FOR QUADRATIC EQUATIONS



Equation:_____

Axis of symmetry:_____

Vertex:_____

Min/Max Value:_____

$y = x^2$

x	y

Vertex point

**NOTE:* We will be using a table on this problem and on Example 1, but we will be using the pattern that you see in the Parent Function and the value of *a* to plot our points after that.

PROPERTIES OF THE GRAPH $y = ax^2 + bx + c$

- The graph opens up if a _____ 0 and down if a _____ 0.
- The graph is vertically stretched if |a| _____ 1 and vertically shrunk if |a| _____ 1.
- The axis of symmetry is x = _____ and the vertex has an x coordinate of _____.
- The y-intercept of the graph is _____. So the point (0, c) is on the parabola.

Identify the Axis of Symmetry, Vertex, and y – Intercept of the following graphs. Also state if that Vertex is a Max or Min.

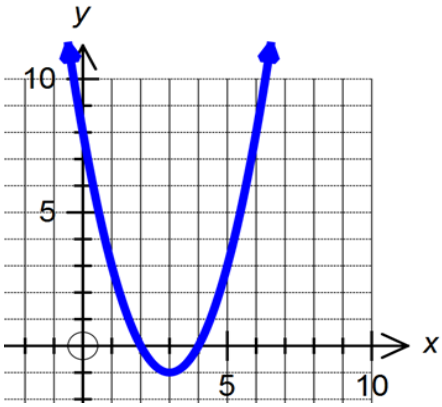
1.

Vertex:

Max or Min

A.O.S.

***y* Intercept**



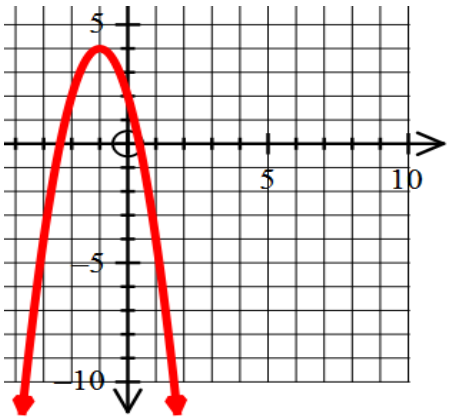
2.

Vertex

Max or Min

A.O.S.

***y* Intercept**

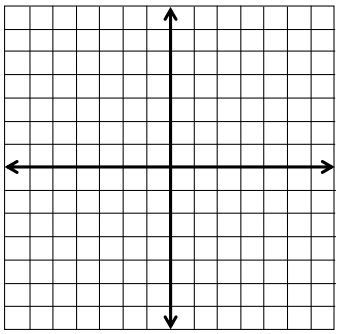


Graph a function in the form of y = ax^2 + bx + c.

3. $y = -x^2 + 4x - 3$

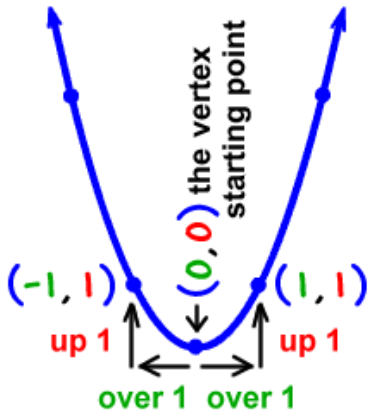
- Identify a, b, c:
- Find the vertex:
- Draw the axis of symmetry:
- Identify the *y*-intercept:
- Complete the table using other *x*-values near the vertex:
- Draw the parabola.

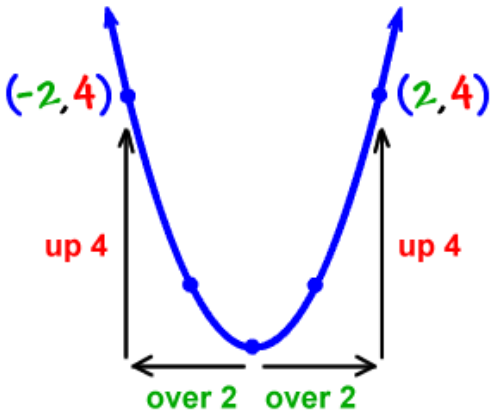
x	*y*



While we could create tables for every quadratic that we want to graph, we can also use the pattern seen in the parent function and the previous problem to graph quadratics. Here's how it works.

First, find the vertex. In the parent function, the vertex was (0, 0). Since a = 1,

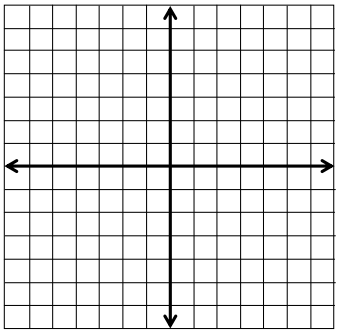




↔		↕
-2	4×1	4
-1	1×1	1
0	0×1	0
1	1×1	1
2	4×1	4

Let's try out this method on this problem. We will use the table to check our work.

4. $y = x^2 - 6x + 5$



Vertex:

Axis of Symmetry:

Min/Max value:

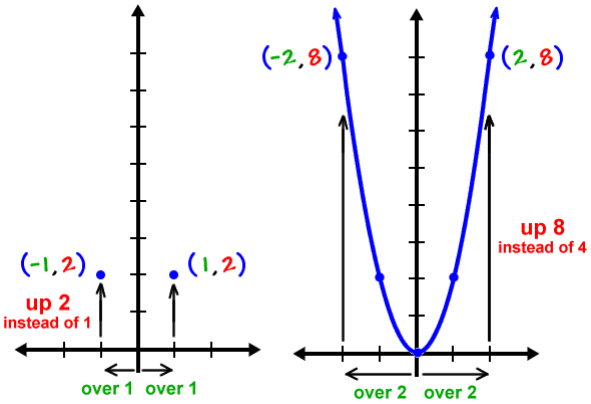
Compare this graph with the parent function.

Comparison:_____

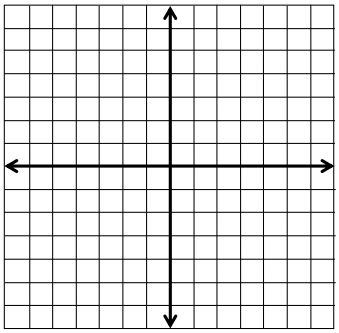
Now, how do we use this method if $a \neq 1$? We need to adjust the up or down movement by multiplying the amount we move up or down by the value of a and then using that point to help graph.

If we have the function $y = 2x^2$, then $a = 2$, which means that the graph of the parent function is vertically stretched by a factor of 2 (or the up movement is twice as big).

<u>Axis of Symmetry Equation</u> $\boldsymbol{x = \frac{-b}{2a}}$



5. $g(x) = -2x^2 + 2$



Vertex:

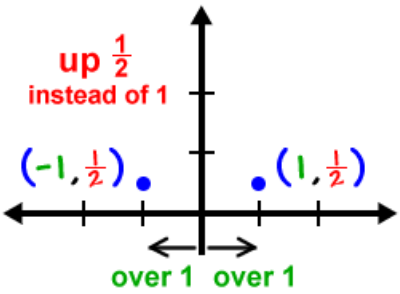
Axis of Symmetry:

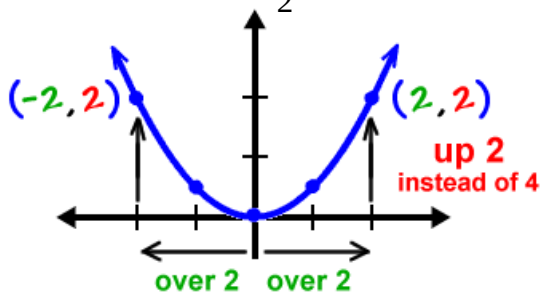
Min/Max value:

Compare this graph with the parent function.

Comparison:_____

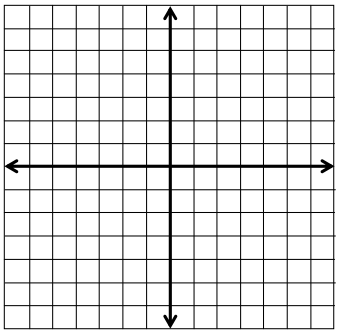
Sometimes, a will be a fraction, but that does not change how we plot points and graph the quadratic. Now, if $|a|$ is less than 1, the graph will be vertically compressed (flattened). If $y = \frac{1}{2}x^2$, then





When you are dealing with fractions, it might be best if you choose to move left or right by a multiple of the denominator in the fraction if possible.

6. $y = -\frac{1}{3}x^2 + 2x - 2$



Vertex:

Axis of Symmetry:

Min/Max value:

Compare this graph with the parent function.

Comparison:_____

MINIMUM AND MAXIMUM VALUES

- If a > 0, then there is a _____ value.
- If a < 0, then there is a _____ value.

<u>Find the minimum or maximum value.</u>

5. $y = -3x^2 + 12x - 6$

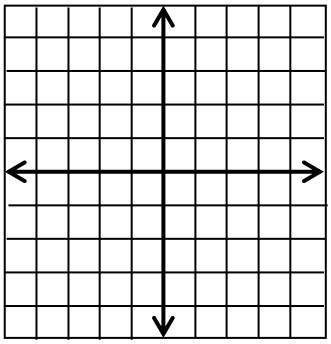
6. $f(x) = 7x^2 + 21x + 8$

Day 2 – Graph Quadratic Functions in Vertex Form

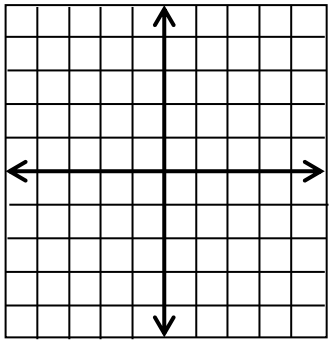
Objectives: Graph functions expressed symbolically by hand and show key features of the graph, including intercepts, vertex, maximum and minimum values, and end behaviors.

Quick review, graph the following absolute value functions:

1. $y = -3|x+2| - 1$



2. $P(x) = \frac{1}{2}|x| - 2$



Vertex form –

VERTEX FORM OF A QUADRATIC FUNCTION: $y = a(x - h)^2 + k$

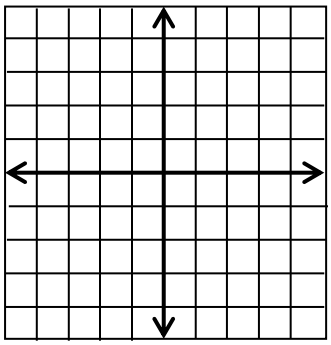
h translates the graph _____ or _____

k translates the graph _____ or _____

a vertically _____ or _____ the graph and if *a* is negative _____

Graph the following quadratic functions in vertex form. Label the vertex and axis of symmetry.

3. $y = (x+1)^2 - 2$

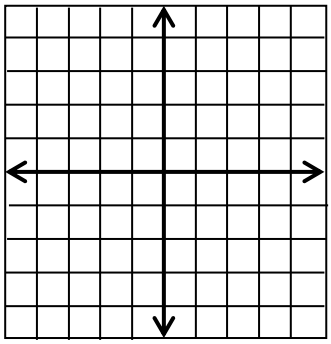


Vertex:

Axis of Symmetry:

Min/Max:

4. $y = -2(x-3)^2 + 4$

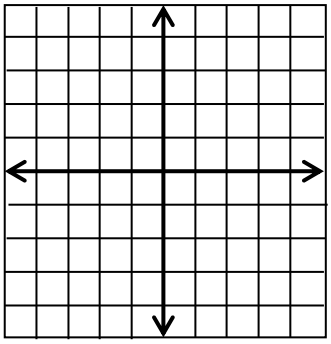


Vertex:

Axis of Symmetry:

Min/Max:

5. $f(x) = \frac{1}{2}x^2 - 3$



Vertex:

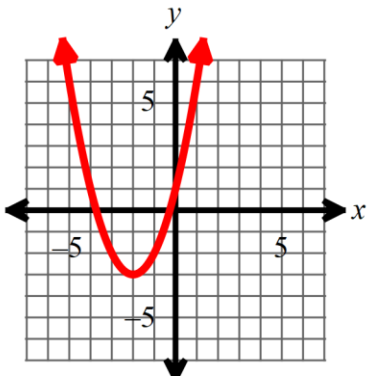
Axis of Symmetry:

Min/Max:

Day 3 – Finding the Domain, Range and End Behavior of Quadratic Functions

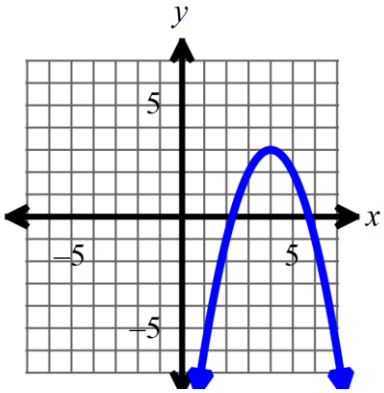
Objectives: Identify and understand the Domain, Range, and End Behavior of quadratic functions.

End Behavior: **Domain:** **Range:**



Given the following graphs, find their end behavior.

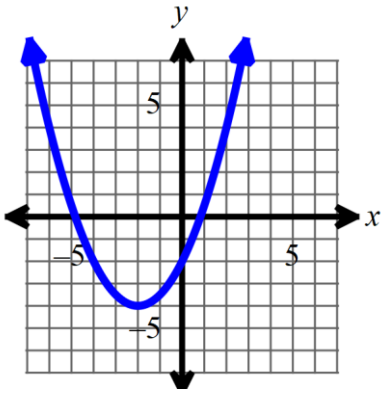
1.



$f(x) \to$ _____ $as\ x \to -\infty\ and$

$f(x) \to$ _____ $as\ x \to +\infty$

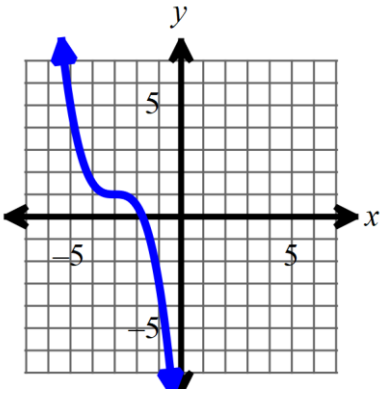
2.



$f(x) \to$ _____ $as\ x \to -\infty\ and$

$f(x) \to$ _____ $as\ x \to +\infty$

3.

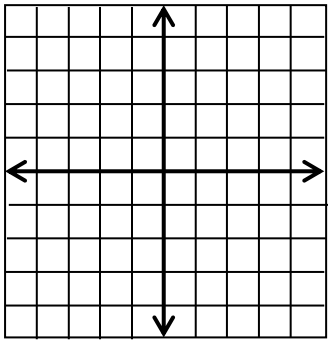


$f(x) \to$ _____ $as\ x \to -\infty\ and$

$f(x) \to$ _____ $as\ x \to +\infty$

Graph the following quadratic functions in vertex form. Label the vertex and axis of symmetry.

3. $y=(x-3)^2-6$



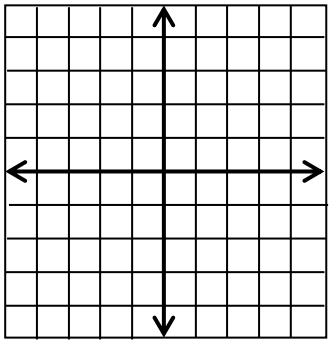
Domain:_____

Range:_____

End Behavior:

$f(x)\to$ _____ $as\ x\to-\infty\ and$
$f(x)\to$ _____ $as\ x\to+\infty$

4. $y=-x^2+2x+1$



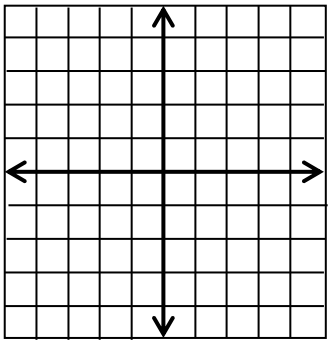
Domain:_____

Range:_____

End Behavior:

$f(x)\to$ _____ $as\ x\to-\infty\ and$
$f(x)\to$ _____ $as\ x\to+\infty$

5. $f(x)=\frac{1}{3}x^2-3$



Domain:_____

Range:_____

End Behavior:

$f(x)\to$ _____ $as\ x\to-\infty\ and$
$f(x)\to$ _____ $as\ x\to+\infty$

Day 5 – Completing the Square

Objectives: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

Perfect Square Trinomial

Find the number that would create a perfect square trinomial, and then write your answer in factored form.

1. $x^2 + 4x +$ _____

2. $y^2 + 10y +$ _____

3. $m^2 - 6m +$ _____

4. $a^2 - 2a +$ _____

5. $c^2 + c +$ _____

6. $k^2 - 9k +$ _____

Complete the square in order to write the equation in vertex form: $y = a(x - h)^2 + k$

7. $y = x^2 + 8x +$ _____ – _____

8. $f(x) = x^2 - 10x +$ _____ – _____

9. $g(x) = x^2 - 2x +$ ____ – _____

10. $y = x^2 - 5x$

11. $y = x^2 + 18x$

12. $h(x) = -x^2 - 24x$

13. $y = -x^2 - 6$

Day 6 – Completing the Square to Graph Quadratic Equations in Vertex Form

Objectives: Use the process of completing the square to graph a quadratic and show the vertex, axis of symmetry, and the minimum or maximum value of the quadratic.

Using Completing the Square to write equations in the form: $y = a(x - h)^2 + k$

- Group x^2 and x terms leaving out the constant term
- Find the number which completes the square.
- Add the value inside the parenthesis and subtract it outside the parenthesis
- Write in factored form and state the vertex

Write in vertex form and then state the vertex.

1. $y = x^2 + 8x + 6$

Vertex:_____

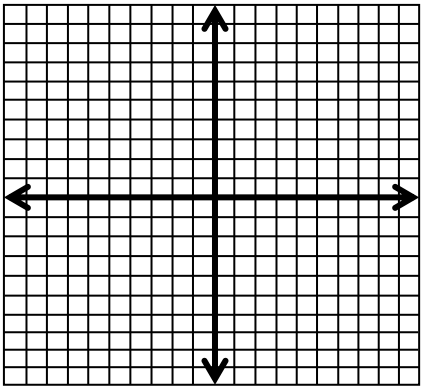
2. $f(x) = -x^2 + 12x - 9$

Vertex:_____

3. $y = x^2 + 8x + 7$

Vertex:_____

4. $y = x^2 - 6x + 3$



Vertex:_____

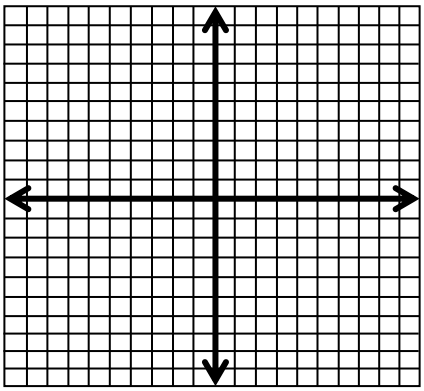
Domain:_____

Range: _____

End Behavior:

$f(x) \to$ _____ $as\ x \to -\infty\ and$
$f(x) \to$ _____ $as\ x \to +\infty$

5. $y = -x^2 + 2x + 1$



Vertex:_____

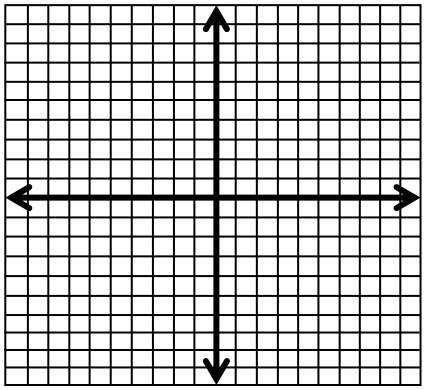
Domain:_____

Range: _____

End Behavior:

$f(x) \to$ _____ $as\ x \to -\infty\ and$
$f(x) \to$ _____ $as\ x \to +\infty$

6. $j(x) = -x^2 + 6x - 5$



Vertex:_____

Domain:_____

Range: _____

End Behavior:

$f(x) \to$ _____ $as\ x \to -\infty\ and$
$f(x) \to$ _____ $as\ x \to +\infty$

Day 7 – Graph Quadratic Functions in Intercept Form

Objectives: Use the process of Factoring to graph a quadratic and show the important parts of the quadratic.

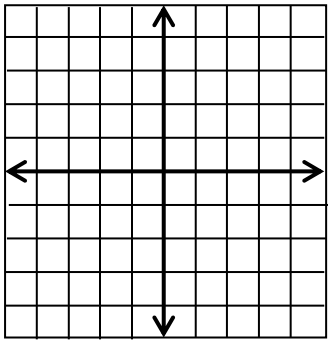
Intercept form –

CHARACTERISTICS OF A GRAPH IN INTERCEPT FORM: $y = a(x - p)(x - q)$

- The x-intercepts are _____ and _____.
- The axis of symmetry is halfway between (p, 0) and (q, 0). It has an equation $x = \frac{\quad}{2}$.
- The same properties of a apply.

Graph the following quadratic functions in intercept form. Label the vertex, the axis of symmetry, state the domain and range, and identify the end behavior of the graphs.

1. $f(x) = (x-4)(x+2)$



Vertex:

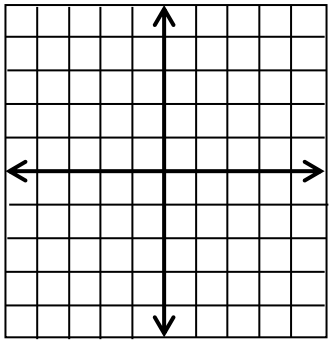
Axis of Sym:

Domain:

Range:

$f(x) \to$ ___ as $x \to -\infty$ and $f(x) \to$ ___ as $x \to \infty$

2. $g(x) = -(x-1)(x-5)$



Vertex:

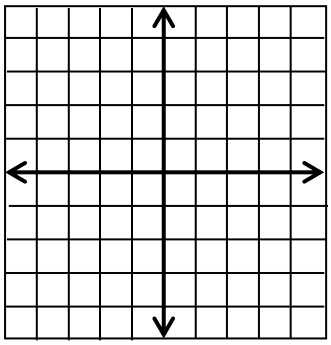
Axis of Sym:

Domain:

Range:

$g(x) \to$ ___ as $x \to -\infty$ and $g(x) \to$ ___ as $x \to \infty$

3. $f(x) = x^2 + 2x - 3$



Vertex:

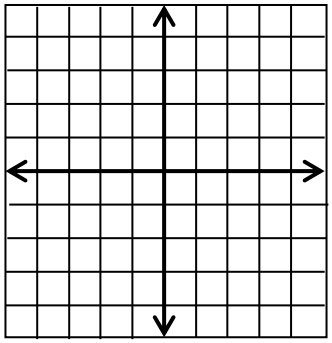
Axis of Sym:

Domain:

Range:

$f(x) \to$ ___ as $x \to -\infty$ and $f(x) \to$ ___ as $x \to \infty$

4. $g(x) = -2x^2 - 8x - 6$



Vertex:

Axis of Sym:

Domain:

Range:

$g(x) \to$ ___ as $x \to -\infty$ and $g(x) \to$ ___ as $x \to \infty$

Day 8 – Changing Forms of a Quadratic Function

Objectives: Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.

	Standard Form	Intercept Form	Vertex Form
What does it look like?			
How do I get it?			

Write the following functions in standard form.

1. $f(x) = -5(x + 2)^2 + 8$
2. $y = 4(x - 3)^2 - 10$
3. Has a vertex at (5, 0) and is vertically stretched by a factor of 2.

Write the following functions in standard form.

4. $y = 3(x + 2)(x - 5)$
5. $f(x) = -3(x - 7)(x + 6)$
6. Has roots of –8 and 3 and is reflected across the x-axis.

Write the following functions in intercept form.

7. $f(x) = (x - 4)^2 - 1$
8. Has a vertex at (–2, –4) and whose end behavior approaches positive infinity.

Write the following functions in vertex form.

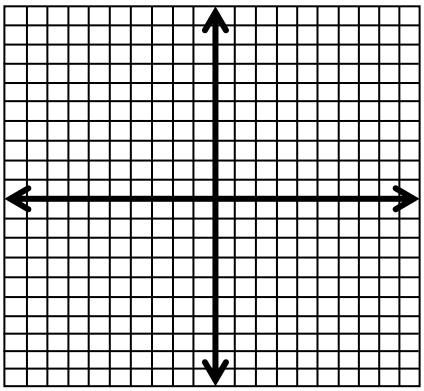
9. $y = (x - 7)(x - 1)$
10. Has zeros of –1 and –5, is vertically shrunk by a factor of 1/2, and is reflected across the x-axis.

Day 9 – Quadratic Equations – Application Day 1

Objectives: Use the properties of quadratics to solve real world problems.

1. Mr. Worthen and Mrs. Worthen have recently taken up the game of tennis. Mr. Worthen lobs his meanest shot to Mrs. Worthen and it can be modeled by the function $f(x) = -\frac{1}{2}(x-3)^2 + 8$ where x is the horizontal distance (in feet) from where Mr. Worthen hit the ball and f(x) is the height of the ball (in feet) above the court.

A) Graph the function and state the domain and range.



Domain:_____ Range:_____

B) In the context of this problem…
What does the domain stand for?
What is it?

C) In the context of this problem…
What does the range stand for?
What is it?

D) What is the maximum height of the tennis ball?

E) From what height is the tennis ball hit?

F) How far away from Mr. Worthen does the tennis ball reach its maximum height?

G) If Mrs. Worthen is blinded by the sun, and does not return the fierce lob hit by Mr. Worthen, how far does Mr. Worthen's shot go?

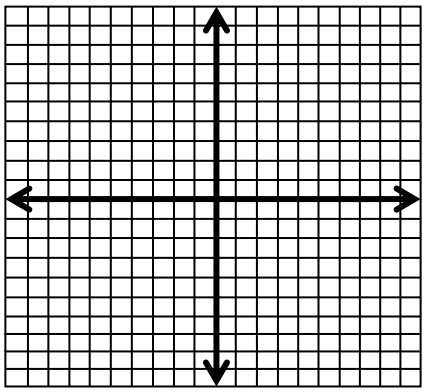
H) If Mr. Worthen is six feet away from the net, which has a height of 3 feet, will the ball even clear the net? Explain your reasoning.

I) What does $f(1) = 6$ represent in terms of this problem?

J) What is the height of the ball after it has traveled 5 feet horizontally?

2. On the court next to Mr. Worthen, Mr. Liessmann and Ms. Babbitt are also partaking in a feisty game of tennis. Ms. Babbitt's latest shot can be modeled by the function $h(t) = -1.5t^2 + 3t + 4.5$ where t is the time (in seconds) that the ball is in the air and $h(t)$ is the height of the ball (in feet) above the court.

A) Graph the function and state the domain and range.



Domain:_____ Range:_____

B) In the context of this problem…
What does the domain stand for?
What is it?

C) In the context of this problem…
What does the range stand for?
What is it?

D) At what time does the tennis ball reach its maximum height?

E) What is the maximum height?

F) From what height is the tennis ball hit?

G) If Mr. Liessmann slips and does not return the shot hit by Ms. Babbitt, how far does the ball go?

H) So if the ball is not hit, how long was it in the air for when it hits the ground?

I) What does $h(2.5) = 2.625$ represent in terms of this problem?

3. Both of the games above though pale in comparison to the match taking place between Mr. Mattick and Mr. Mortara on Centre Court. Mr. Mortara zings a shot in Mr. Mattick's direction, and he dives to barely hit it a split second before it hits the ground. If Mr. Mattick's shot is modeled by the following function

$y = -.0025x(x - 78.9)$ where x is the horizontal distance (in feet) from where Mr. Mattick hit the ball and y is the height of the ball (in feet) above the court, does Mr. Mattick's shot land inside the court if he is 78 feet away from Mr. Mortara's baseline?

4. The Tacoma Narrows Bridge in Washington has two towers that each rise 307 feet above the roadway and are connected by suspension cables. Each cable can be modeled by the function $f(x) = \frac{1}{7000}(x - 1400)^2 + 27$, where x is the distance from one tower in feet and $f(x)$ is the height of the cable above the roadway. What is the distance between the two towers?

5. According to the function in problem #4, how far above the roadway is the cable at its lowest point?

6. The path of a kicked soccer ball can be modeled by the function $y = -0.02x(x - 80)$, where x is the horizontal distance in feet and y is the height of the ball in feet. How far does the soccer ball travel in the air?

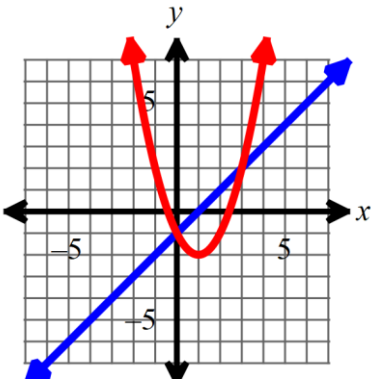
7. Using the information about the soccer ball above, what is the maximum height of the ball?

Day 10 – Solving Systems of Linear and Quadratic Functions / Inequalities

Objectives: Be able to solve systems of equations graphically and algebraically; and solve systems of inequalities graphically.

System of two equations

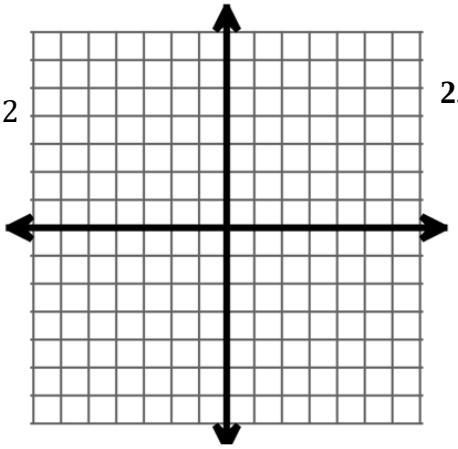
Solution of a system



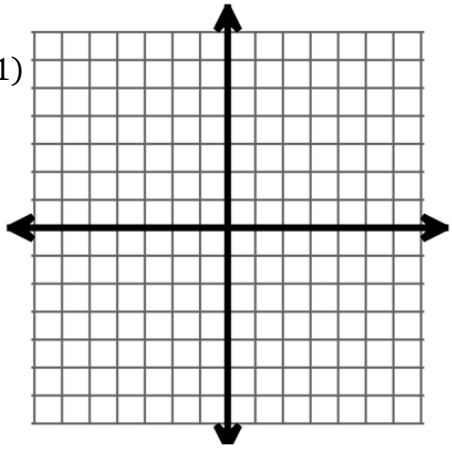
Solving the systems graphically

- Graph both equations individually.
- Find the intersection point.
- Answer is the x-coordinates of the intersection points.

1. $\begin{cases} y = x - 1 \\ y = (x-1)^2 - 2 \end{cases}$



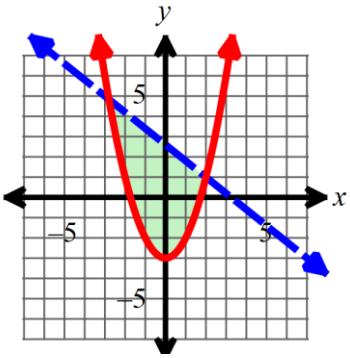
2. $\begin{cases} y = \frac{1}{2}(x+3)(x-1) \\ y = (x+1)^2 - 4 \end{cases}$



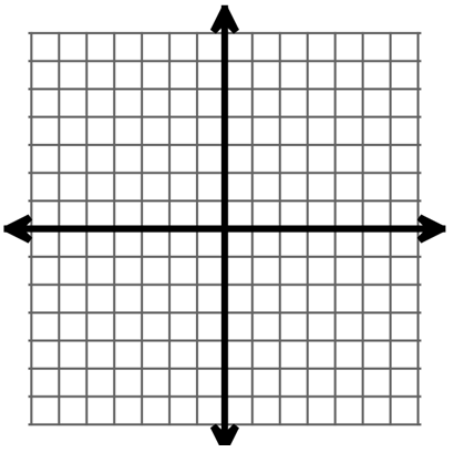
Solving a System of Inequalities

Solid Line vs. Dotted Line

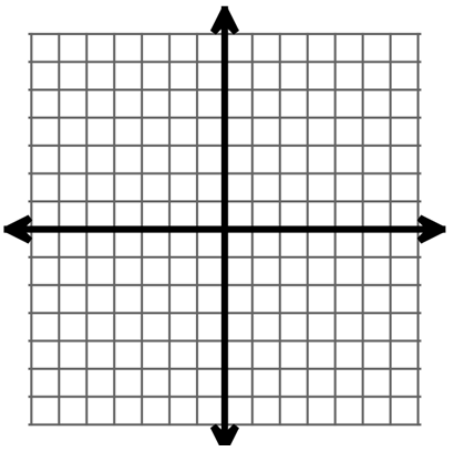
Solution to a System of Inequalities



3. $\begin{cases} y < 2x - 3 \\ y \geq -(x-1)^2 + 3 \end{cases}$



4. $\begin{cases} x + 1 \geq 0 \\ y \geq x^2 - 4 \end{cases}$



<u>Solving a System of Equations Algebraically</u>

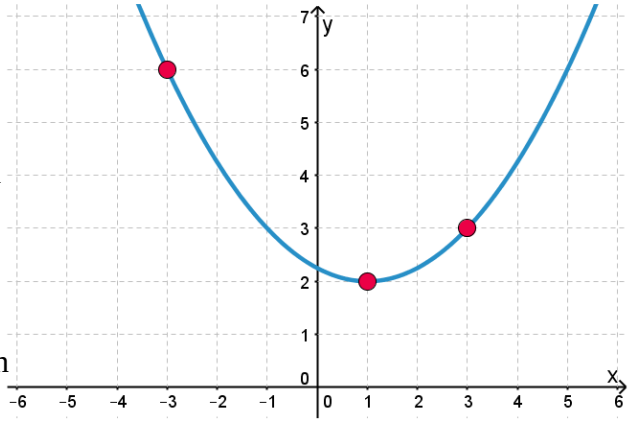
- Set both equation equal to one variable (usually y)
- Set equations equal to each other
- Solve for x

5. $\begin{cases} y = x^2 - 2x + 2 \\ y - 2x = -2 \end{cases}$

6. $\begin{cases} y = 2x^2 - 2x + 3 \\ y = x^2 + 5x - 7 \end{cases}$

Day 11 – Writing the Equations of Parabolas using 3 points

Objectives: Write equations of parabolas using 3 points.



Fitting a parabola to three points using Matrices.

- Plug each of the three points into x and y of the standard form equation $y = ax^2 + bx + c$ – this will give you an a, b, c equation
- Put the three equations into a matrix, and solve using Algebra or QuadReg (Calculator)
- Once you have solved for a, b, c – re-write your equation in standard form

<u>Write a matrix equation for in standard form for the parabola passing through the points.</u>

1. $(0,1), (1,5)$ and $(2,3)$

2. $(-2,-24), (0,-6)$, and $(3,-9)$

<u>Write the equation of a parabola in standard from given the following three points.</u>

3. $(-1,9), (1,3)$ and (5.39)

4. $(0,2), (4,7)$ and $(8,28)$

5. Write the equation of parabola through the points $(-1,17), (1,5)$, and $(3,17)$ in **Vertex Form.**