Unit 8 – Rational Functions – Algebra II

Day 1 – Simplifying Rational Expressions

OBJECTIVES: Reduce Rational functions

Reducing Fraction Review

1. $\frac{30}{42}$ **2.** $\frac{15}{45}$ **a.** $\frac{72}{36}$

Factoring Review

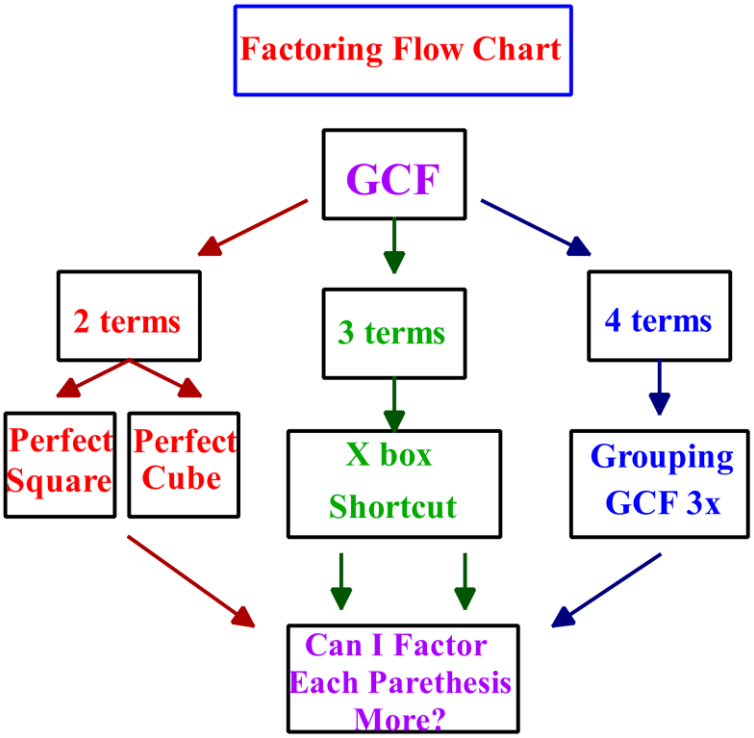
Terms

GCF

X-box vs Shortcut

Perfect Squares / Cubes

Grouping



Factor the following.

3. $6x-15$ **4.** x^2-49 **5.** $x^2+7x+12$ **6.** $3x^2-19x+6$

Simplified Form of a Rational Expression Cancel Out

Simplify the following rational expressions.

7. $\frac{4+6}{4}$ **8.** $\frac{3x+12}{3}$ **b.** $\frac{15x-5}{10}$

Before you begin, please put _____ around the _____ and the _____.

9. $\frac{4y^2+8y}{y^2-4}$

	Number of Terms	Factoring Method	Factor	Rewrite and Cancel Out	Double Check
Top					
Bottom	**Number of Terms**	**Factoring Method**	**Factor**		**Answer**

10. $\dfrac{-4x-20}{x^2+11x+30}$

	Number of Terms	Factoring Method	Factor	Rewrite and Cancel Out	Double Check
Top					
Bottom	**Number of Terms**	**Factoring Method**	**Factor**		**Answer**

c. $\dfrac{x^2+3x-10}{3x^2-6x}$

Factor Top		**Rewrite and Cancel Out**		**Factor Top**		**Rewrite and Cancel Out**	
Factor Bottom				**Factor Bottom**			
Answer				**Answer**			

12. $\dfrac{x^2-1}{x^2-2x+1}$

13. $\dfrac{x-3}{x^3-27}$

Perfect Cube Rules

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

14. $\dfrac{2x^2+7x-4}{8x^2-4x}$

15. $\dfrac{x^3+3x^2-4x-12}{x^2+5x+6}$

Day 2 – Multiply Rational Expressions

OBJECTIVES: Reduce Rational functions and Multiply Rational Functions

Multiplying Fraction Review

1. $\frac{4}{5} \bullet \frac{15}{16}$

2. $\frac{4}{15} \bullet 5$

a. $\frac{12}{5} \bullet \frac{25}{16}$

Monomial vs Polynomial

Multiplying Monomial Expressions

Terms

4. $\frac{x^2 y}{xy^2} \bullet \frac{y^3}{2xy}$

5. $\frac{5a^2}{3b^2} \bullet \frac{9a^2 b}{15ab}$

b. $\frac{11x^7}{4y^6} \bullet \frac{64x^3 y}{44xy}$

Multiplying Rational Expressions with Multiple Terms

6. $\frac{x+4}{3(x-5)} \bullet \frac{6x}{x(x+4)}$

7. $\dfrac{5k-10}{k^2+3k-10} \bullet \dfrac{9k^2}{6k^2-12k}$

8. $\dfrac{5y^2-10y}{2y^2-2} \bullet \dfrac{6y^2-6}{15y-30}$

FACTOR EACH PART		FACTOR EACH PART	
Factor	**Factor**	**Factor**	**Factor**
Factor	**Factor**	**Factor**	**Factor**
RE-WRITE AND CANCEL OUT		**RE-WRITE AND CANCEL OUT**	

7. $\dfrac{3y^2-12}{12y+120} \bullet \dfrac{4y^2-400}{y^2+8y-20}$

c. $\dfrac{x-4}{x^3+1} \bullet \left(x^2-x+1\right)$

FACTOR EACH PART		FACTOR EACH PART	
Factor	**Factor**	**Factor**	**Factor**
Factor	**Factor**	**Factor**	**Factor**
RE-WRITE AND CANCEL OUT		**RE-WRITE AND CANCEL OUT**	

8. $\dfrac{-2x^4+12x^3-18x^2}{x^3-27}\bullet\left(x^2+3x+9\right)$

Perfect Cube Rules

$$a^3+b^3=(a+b)(a^2-ab+b^2)$$

$$a^3-b^3=(a-b)(a^2+ab+b^2)$$

Day 3 – Dividing Rational Functions

OBJECTIVES: Dividing Rational functions

Dividing Fraction Review

1. $\frac{3}{8} \div \frac{9}{16}$

2. $10 \div \frac{1}{2}$

a. $\frac{16}{45} \div \frac{8}{5}$

Dividing Monomial Rational Expressions

Simplify the following rational expressions.

3. $\frac{6a^2}{8b} \div \frac{12a^3}{16ab^2}$

b. $\frac{5a^2b^3}{a^7} \div \frac{30ab^4}{b^3}$

Dividing Rational Expressions with Multiple Terms

5. $\dfrac{7x^2}{x^2-3x} \div \dfrac{21x}{x^2+4x-21}$

FACTOR EACH PART		REWRITE
Factor	**Factor**	
Factor	**Factor**	
FLIP		**CANCEL OUT**

6. $\dfrac{5y+20}{20y} \div \left(y^2-16\right)$

FACTOR EACH PART		REWRITE
Factor	**Factor**	
Factor	**Factor**	
FLIP		**CANCEL OUT**

c. $\dfrac{8}{x+7} \div \dfrac{8x^2-8x}{x^2+6x-7}$

FACTOR EACH PART		REWRITE
Factor	**Factor**	
Factor	**Factor**	
FLIP		**CANCEL OUT**

7. $\dfrac{k^2-m^2}{k^2-2km-3m^2} \div \dfrac{6k-6m}{3}$

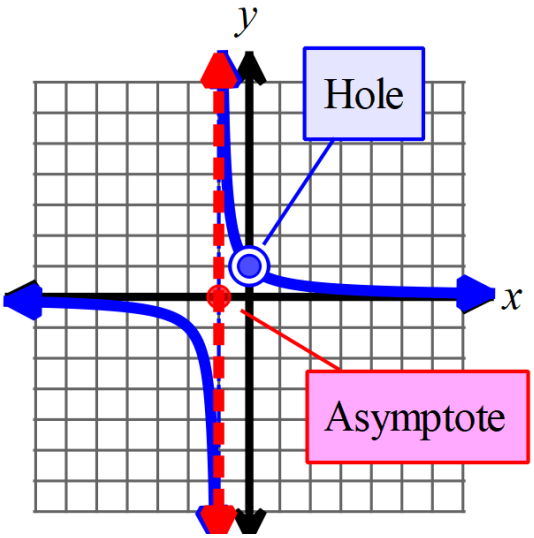
8. $\dfrac{k-5}{9k^2-18k} \div \dfrac{2k^2-11k+5}{2k^2-5k+2}$

Day 4 – Domain of Rational Expressions and Undefined Values

Objectives: Find the Domain restriction of a Rational Expressions

Asymptotes

Holes



Domain

Domain is undefined: the domain of a function is undefined when the _____ is _____ because you can NOT divide anything by _____.

For what value(s) of the variable is the expression undefined?

1. $\frac{x^2+4}{x}$

2. $\frac{1}{x-3}$

3. $\frac{6}{x^2-x-42}$

Find the domain of each of the following functions – determine if the undefined part is a hole or an asymptote.

4. $y=\frac{5}{2x-1}$

5. $f(x)=\frac{2x}{x+5}$

6. $y=\frac{4x}{x(x-3)}$

7. $f(x) = \dfrac{-3}{x^2 - 4x}$

8. $y = \dfrac{3x^2}{x^2 - 4}$

Day 5 – Adding and Subtracting Rational Functions – Part 1 – Common Denominators

OBJECTIVES: Adding and Subtracting Rational Functions

Adding and Subtraction Fractions with Common Denominators

Simplify the following rational expressions.

1. $\frac{11}{12}+\frac{7}{12}$

2. $\frac{a}{6}+\frac{5a}{6}$

a. $\frac{18}{6}-\frac{15}{6}$

3. $\frac{3a+1}{4}+\frac{a-5}{4}$

4. $\frac{6y+10}{5}-\frac{y}{5}$

b. $\frac{3a+1}{4}-\frac{a-5}{4}$

5. $\frac{3m}{m-7}-\frac{21}{m-7}$

6. $\frac{9x^2}{3x+1}-\frac{1}{3x+1}$

Adding and Subtraction Fractions with Uncommon Denominators (But Super Close)

9. $\frac{3a}{a-4}-\frac{a-16}{4-a}$

10. $\frac{y^2}{y-1}+\frac{1}{1-y}$

11. $\frac{x^2-3x}{x-5}+\frac{10}{5-x}$

12. $\frac{3y+1}{y^2-16}-\frac{2y-3}{y^2-16}$

Day 6 – Adding and Subtracting Rational Functions – Part 2 – Uncommon Denominators

OBJECTIVES: Adding and Subtracting Rational Functions

Adding and Subtraction Fractions with Uncommon Denominators

LCD

Simplify the following rational expressions.

1. $\frac{5}{6}-\frac{1}{3}$

2. $\frac{a}{3}+\frac{a}{6}$

3. $\frac{5}{3x}-\frac{3}{4}$

4. $\frac{4}{4m}+\frac{-6}{m^2}$

LCD

5. $\frac{x}{x+1}-\frac{1}{x}$

LCD

6. $\frac{m}{m+2} - \frac{4}{m-2}$

LCD

7. $\frac{x+1}{x+4} - \frac{24}{x^2-16}$

LCD

8. $\frac{x+1}{x^2+6x+9} + \frac{6}{x^2-9}$

LCD

9. $\frac{x-5}{x^2-6x+5} \bullet \frac{2x-2}{4x^2}$

LCD

Day 7 – Complex Fractions – Putting it all together

OBJECTIVES: Simplify Complex Fractions and Rational Expressions

Complex Fractions

Simplifying Complex Fractions

Simplify the following Complex fractions.

1. $\dfrac{3/5}{9/10}$

2. $\dfrac{\frac{1}{2}+\frac{2}{3}}{\frac{3}{4}}$

COMBINE TOP	REWRITE	FLIP	MULTIPLY AND REDUCE
COMBINE BOTTOM			

3. $\dfrac{4}{2+\dfrac{1}{x}}$

COMBINE TOP	REWRITE	FLIP	MULTIPLY AND REDUCE
COMBINE BOTTOM			

4. $\dfrac{\dfrac{x}{3}+6}{10-\dfrac{4}{x}}$

COMBINE TOP	REWRITE	FLIP	MULTIPLY AND REDUCE
COMBINE BOTTOM			

5. $$\frac{\dfrac{16}{x-2}}{\dfrac{4}{x+1}-\dfrac{6}{x}}$$

6. $$\frac{\dfrac{3}{x-2}-\dfrac{6}{x^2-4}}{\dfrac{3}{x+2}+\dfrac{1}{x-2}}$$

Day 8 – Solving Rational Functions – Cross Multiplication Method

OBJECTIVES: Solve Rational Functions

Cross Product Property

Extraneous Solutions:

Solve each equation by cross multiplying. Check for *extraneous* solutions!

1. $\frac{x}{15} = \frac{16}{60}$

2. $\frac{x}{4} = \frac{6}{19}$

3. $\frac{2x}{3} = \frac{3}{18}$

4. $\frac{x+1}{x} = \frac{6}{12}$

Which numbers
Can't be solutions?

_____ , _____

5. $\frac{3}{5x} = \frac{2}{x-7}$

Which numbers
Can't be solutions?

_____ , _____

a. $\frac{-4}{x+3} = \frac{5}{x-3}$

Which numbers
Can't be solutions?

_____ , _____

6. $\frac{1}{2x+5}=12$

Which numbers
Can't be solutions?

_____

7. $\frac{x-3}{x+5}=\frac{x}{x+2}$

Which numbers
Can't be solutions?

_____ , _____

b. $\frac{5}{x-4}=\frac{x}{2x-8}$

Which numbers
Can't be solutions?

_____

8. $\frac{9}{x^2-6x+9}=\frac{3x}{x^2-3x}$

9. $\frac{4(x-4)}{x^2+2x-8}=\frac{4}{x+4}$

Day 9 – Solving Rational Functions – Part 2

OBJECTIVES: Solve Rational Functions

Denominator Elimination Method

Solve each equation by eliminating the common denominator. Check for *extraneous* solutions!

1. $\frac{x}{3}-\frac{1}{2}=\frac{3x+1}{6}$

Lowest Common Denominator							Rewrite Numerators Only
Frac #1		Frac #2		Frac #3			
Missing		Missing		Missing			Solve
Multiply to Top and Bottom		Multiply to Top and Bottom		Multiply to Top and Bottom			

2. $\frac{5}{x}+\frac{7}{4}=-\frac{9}{x}$

Lowest Common Denominator							Rewrite Numerators Only
Frac #1		Frac #2		Frac #3			
Missing		Missing		Missing			Solve
Multiply to Top and Bottom		Multiply to Top and Bottom		Multiply to Top and Bottom			

3. $\frac{5}{x}-\frac{1}{3}=\frac{1}{x}$

Lowest Common Denominator							Rewrite Numerators Only
Frac #1		Frac #2		Frac #3			
Missing		Missing		Missing			Solve
Multiply to Top and Bottom		Multiply to Top and Bottom		Multiply to Top and Bottom			

a. $\frac{2}{3}-\frac{1}{x}=\frac{5}{6}$

Lowest Common Denominator						Rewrite Numerators Only
Frac #1		Frac #2		Frac #3		
Missing		Missing		Missing		Solve
Multiply to Top and Bottom		Multiply to Top and Bottom		Multiply to Top and Bottom		

4. $\frac{3}{2}+\frac{4}{x-1}=\frac{x+1}{x-1}$

Lowest Common Denominator						Rewrite Numerators Only
Frac #1		Frac #2		Frac #3		
Missing		Missing		Missing		Solve
Multiply to Top and Bottom		Multiply to Top and Bottom		Multiply to Top and Bottom		

5. $1-\frac{8}{x-5}=\frac{3}{x}$

6. $\frac{6}{x-3}=\frac{4x^2}{x^2-9}-\frac{4}{x+3}$

7. $\frac{x}{x-1}-\frac{1}{x-2}=\frac{2x-5}{x^2-3x+2}$

8. $\frac{5}{x}-2=\frac{2}{x+3}$

Day 11 – Inverse Variation

Objectives: SWBAT write direct, inverse variation equations

Direct Variation

Inverse Variation

Joint Variation

Constant of Variation

Determine if *x* and *y* vary directly, inversely, or neither.

1) $xy = 7$

2) $y = 2x + 3$

a) $\frac{y}{5} = x$

3) $\frac{y}{x} = 8$

4) $x - y = 10$

b) $y = \frac{x}{12}$

Write an equation for the following where *x* and *y* vary inversely. Then find y when $x = -2$.

5) $x = 4, and\ y = 3$

6) $x = 8, and\ y = -1$

c) $y = -\frac{1}{2}, and\ x = 4$

7) If x and y vary inversely and $x = 1.5$ when $y = 8$, what is the value of y when $x = -20$?

<u>Determine if the following tables represent an inverse variation.</u>

8)

x	1	2	3	4	6	12
y	12	6	4	3	2	1

9)

x	1	2	3	4	5	6
y	20	17	14	11	8	5

d)

x	2	4	6	8	10	12
y	30	15	10	7.5	6	5