

Precalculus Formulas for Final Exam

Arithmetic: **Specific term:** $a_n = a_1 + (n-1)d$

Sum of series: $S_n = \frac{n}{2}(a_1 + a_n)$ or $S_n = \frac{n}{2}(2a_1 + (n-1)d)$

Geometric: **Specific term:** $a_n = a_1 \cdot r^{n-1}$

Sum of series: $S_n = a_1 \left(\frac{1-r^n}{1-r} \right)$ or $\frac{a_1(1-r^n)}{(1-r)}$

Infinite Sum: $S_\infty = \frac{a_1}{1-r}$ when $-1 < r < 1$ $r \neq 0$

A Formula for Expanding Binomials: The Binomial Theorem

For any positive integer n ,

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \binom{n}{3}a^{n-3}b^3 + \dots + \binom{n}{n}b^n$$

$$= \sum_{r=0}^n \binom{n}{r} a^{n-r} b^r.$$

Pascal's Triangle

			1			
		1	1	1		
	1	1	2	1		
	1	3	3	1		
1	4	6	4	1		
1	5	10	10	5	1	
1	6	15	20	15	6	1
1	7	21	35	35	21	7
1						1

Definition of Derivative

Slope of the Tangent Line to a Curve at a Point

The **slope of the tangent line** to the graph of a function $y = f(x)$ at $(a, f(a))$ is given by

$$m_{\tan} = \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h}$$

provided that this limit exists. This limit also describes

- the **slope of the graph of f at $(a, f(a))$** .
- the **instantaneous rate of change of f with respect to x at a** .

Ellipse

Remember $(a^2 - b^2 = c^2)$

Hyperbola

Remember $(a^2 + b^2 = c^2)$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

Parabola

$$(y-k)^2 = 4p(x-h)$$

Or

$$(x-h)^2 = 4p(y-k)$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

$$x^2 + y^2 = r^2$$

$$\tan^{-1}\left(\frac{y}{x}\right) = \theta$$

$$\text{Angle between } \theta = \cos^{-1}\left(\frac{\mathbf{v} \cdot \mathbf{w}}{\|\mathbf{v}\| \|\mathbf{w}\|}\right)$$

DeMoivre's Thm.

$$(r \operatorname{cis}(\theta))^n = r^n \operatorname{cis}(n\theta) \quad \text{or} \quad [r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta)$$

Product of Two Complex Numbers in Polar Form

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ be two complex numbers in polar form. Their product, $z_1 z_2$, is

$$z_1 z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + i \sin(\theta_1 + \theta_2)].$$

To multiply two complex numbers, multiply moduli and add arguments.

Quotient of Two Complex Numbers in Polar Form

Let $z_1 = r_1(\cos \theta_1 + i \sin \theta_1)$ and $z_2 = r_2(\cos \theta_2 + i \sin \theta_2)$ be two complex numbers in polar form. Their quotient, $\frac{z_1}{z_2}$, is

$$\frac{z_1}{z_2} = \frac{r_1}{r_2} [\cos(\theta_1 - \theta_2) + i \sin(\theta_1 - \theta_2)].$$

To divide two complex numbers, divide moduli and subtract arguments.

Other 'I should know' stuff:

$$\cos(2\theta) = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$$

$$a^2 = b^2 + c^2 - 2bc \cos A \quad \text{or} \quad A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right)$$

$$\text{Area} = \sqrt{s(s-a)(s-b)(s-c)} \quad s = \frac{a+b+c}{2}$$

Or

$$\text{Area} = \frac{1}{2} * b * c * \sin A$$

$$\sec \theta \cos \theta = 1 \quad \sin \theta \csc \theta = 1 \quad \cot \theta \tan \theta = 1$$

$$1 + \cot^2 \theta = \csc^2 \theta \quad 1 + \tan^2 \theta = \sec^2 \theta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta \quad \cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2\cos^2 \theta - 1 = 1 - 2\sin^2 \theta$$

$$\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} \quad \sin \frac{\theta}{2} = \pm \sqrt{\frac{1 - \cos \theta}{2}} \quad \cos \frac{\theta}{2} = \pm \sqrt{\frac{1 + \cos \theta}{2}} \quad \tan \frac{\theta}{2} = \frac{1 - \cos \theta}{\sin \theta} = \frac{\sin \theta}{1 + \cos \theta}$$