PCT Semester 2 Final Review

Name _____ Date _____

CHAPTER 5:

5.1 – 5.3 PROBLEMS

1. Simplify: sinx·cotx·secx

2. Verify: $\frac{1-\sin^2 x}{\frac{1}{2}\cot x} = \sin 2x$

3. Verify: $\frac{\sec^2 x}{\tan^2 x} = \cot^2 x + 1$

4. If $sinx = (-2/9)$ and x is in Quadrant III, find tan (x)

5. If $cosx = (1/6)$ in QIV, find $\cot^2 x$

6. Simplify: $\sin(x - 3\pi/2)$

7.If sin a = 1/3 in Quadrant II and cos b = 1/5 in Quadrant I, then cos(a − b) is:

7a. Find the exact value: $\cos\left(\frac{7\pi}{12}\right)\sin\left(\frac{5\pi}{12}\right) - \cos\left(\frac{5\pi}{12}\right) sin\left(\frac{7\pi}{12}\right)$

7b. Power Reduce: $8\sin^4(\theta)$

7c. Find the exact value (not a decimal) for $\frac{\tan(25)+\tan(35)}{1-\tan(25)\tan(35)}$

<u>Problems 8 − 10: Solve the given equations for x over the indicated intervals.</u>

8. $6\cos x = \sqrt{27}$, $0 \le x < 2\pi$

9. $2\sin x + 5 = 6\sin x + 7$, $0 \le x < 2\pi$

10. $1 + 2\tan x = 5$, $0° \le \theta < 360°$

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11. If $sinx = -4/5$ and x is in IV, find

a) $sin(2x)$ **b)** $cos(2x)$ **c)** $sin(x/2)$

d) $tan(2x)$ **e)** $cos(x/2)$

12. Find the exact value of $\sin 75^0$ using a sum or difference formula.

13. Find the exact value of $\cos(11\pi/12)$ using a half-angle formula.

<u>For 14 – 17, solve each over [0,2π). Be sure you know how to write solutions for all values of x as well.</u>

14. $\sin^2 x - \sin x = 0$

15. $2\cos^2 x + 3\cos x + 1 = 0$

16. $3\tan^2 x - 9\tan x = 0$

17. $4\sin^2 x - 1 = 0$

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5.5 TRIG EQUATIONS

Problems 18-20: Solve the given equations for x over the indicated intervals.

18. $6\cos x = \sqrt{27}$, $0 \le x < 2\pi$

19. $2\sin x + 5 = 6\sin x + 7$, $0 \le x < 2\pi$

20. $1 + 2\tan x = 5$, $0° \le \theta < 360°$

Solve for all values in radians.

21. $\cos^2 x - \cos x = 0$

22. $2\sin^2 x + \sin x - 1 = 0$

23. $\tan(3x) = -1$

24. $\sin(2x) + \cos x = 0$

CHAPTER 6: Vectors and Polar

Polar 6.5

25. Find three other representations of the polar point (3, 60°)

26. A point with rectangular coordinates (−4, 10) has polar coordinates

27. Find three other representations of polar point $(-2, -7\pi/6)$

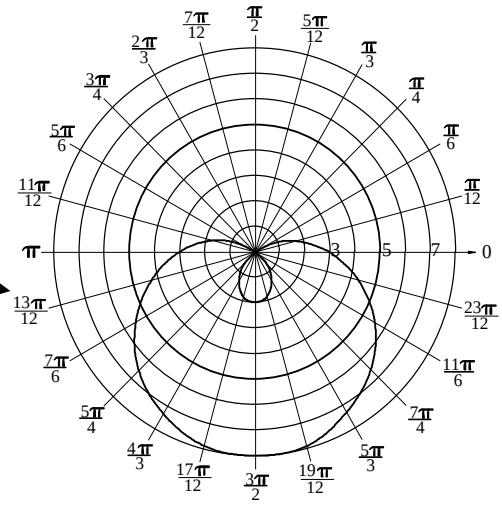
28. Convert $r = -3\csc\theta$ to a rectangular equation.

29. Convert to polar: $x = -25$

30. The graph of $r = 8 - 2\sin\theta$ is a:

31. The graph of $r = 6\sin 7\theta$ is a:

32a). Find the polar equation of the polar graph:



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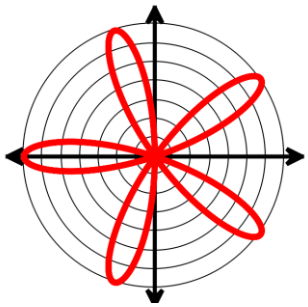
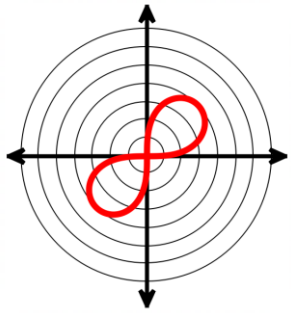
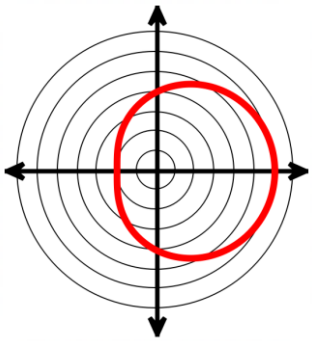
32b)



32c)



33d)



33. Find the polar form of $5 - 3i$.

34. What is the rectangular form of $240 \text{ cis } -105°$?

35. Multiply $\frac{1}{2}(\cos 8° + i \sin 8°)$ and $28(\cos 22° + i \sin 22°)$

36a) Divide $3 \text{ cis } 95°$ by $0.04 \text{ cis } 52°$

36b) Find complex forth roots of $16\left(\cos\left(\frac{\pi}{6}\right) + i\sin\left(\frac{\pi}{6}\right)\right)$

Vectors 6.6 and 6.7

<u>For problems 37 – 39 :</u> $a = 7i - 3j$, $b = -4i + j$, $c = -10i - 2j$

37. Find the vector: $-a + 3b - c$

38a. Find the dot product of vectors a and $(b - c)$

38b) Write the unit vector of $a + b$

39a. What is the magnitude of the vector $c - 3a + b$?

39b) Write as vector in terms of i and j $\|a\| = 8, \theta = 330°$

40. A force of 240 pounds acts at 33°, and a second force of 180 pounds acts at 282°. What is the magnitude and direction of the resultant force?

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41. A plane heads due south at 420 miles per hour. A wind is blowing at N24°E at 72 mph. What is the ground speed and actual direction of the plane?

42. A force of 42 pounds acts at 26°, and another force of 50 pounds acts at 90°. What force at 270° would give an overall resultant force of 48 pounds, and in what direction would that resultant force act?

<u>CHAPTERS 7: Partial Fractions</u>

43a. Find the decomposition of: $\frac{x+4}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2}$

43b. Of $\frac{8x^2+x-17}{(x+1)(x^2-3)}$

43c. $\frac{4x^2+5x-9}{(x-3)(x^2+3x+3)}$

44a. Solve the system: $x^2 + y^2 = 5$ and $2y = x$

44b) $\begin{array}{l} 3x^2 + y^2 = 3 \\ x^2 + y^2 = 1 \end{array}$

44c. Solve the system: $x + y + z = 5$, $2x - y + z = 2$, $3x - y + 2z = 5$

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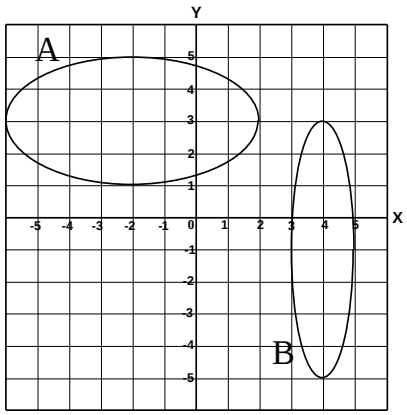
CHAPTER 9 – Conics

45a. Find the equation of ellipse A graphed at the right:



45b. Equation of ellipse B?

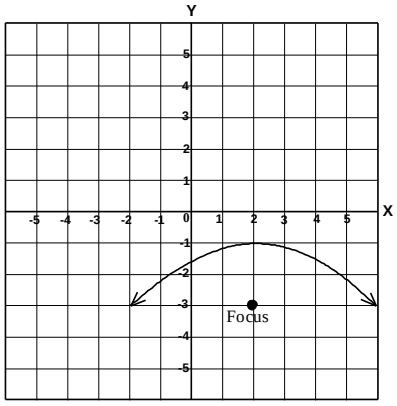
46a. The length of the major axis of the ellipse $\frac{x^2}{1}+\frac{y^2}{49}=1$ is:

46b. Distance between the foci of $\frac{x^2}{14}+\frac{y^2}{22}=1$ is:

47a. An ellipse has vertices of (−4, 0) and (4, 0), and foci at (−2, 0), (2, 0). Find the length of its minor axis.

47b. Ellipse with co-vertices of (0, −6) and (0, 6). Foci at (−5, 0) and (5, 0). Length of its major axis?

48a. The equation of the parabola graphed at the right is:



48b. A parabola has a vertex at (3, −2) and a directrix of x = −5. What is its equation?

49a. The foci of the hyperbola $\frac{x^2}{81}-\frac{y^2}{23}=1$ are:

49b. Foci of $\frac{(y-1)^2}{64}-\frac{x^2}{10}=1$

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50a. Find the vertices of the hyperbola $\frac{x^2}{121} - \frac{y^2}{16} = 1$

50b. Find the vertices Of $\frac{(y+2)^2}{81} - \frac{(x-3)^2}{9} = 1$

51a. A parabola has a focus at (−1, 2) and a directrix of x = 7. What is its equation?

51b. Focus at (220, −80) and a directrix of y = 10. Equation?

51c) Write the equation for the hyperbola in standard form $4x^2 - 24x - 25y^2 + 250y - 489 = 0$

<u>CHAPTER 10 – Sequence and Series</u>

52a. Find the 88st term of the sequence: 7, 10, 13, 16 . . .

52b. 107th term of 30, 26, 22, 18, …

53a. Find the sum of the first 55 terms of the series: $\frac{7}{5} + \frac{4}{5} + \frac{1}{5} - \frac{2}{5} \ldots$

53b. Sum of the first 355 terms of the series: $-7 - \frac{75}{11} - \frac{73}{11} - \frac{71}{11} \ldots$

54a. Find the sum of the infinite series: 8 + 6 + 9/2 + . . .

54b. Sum of $\frac{1}{4} - \frac{5}{24} + \frac{25}{144} - \frac{125}{864} + \ldots$

55a. The sum of the infinite geometric series: $4 + 4r + 4r^2 + 4r^3 + \ldots$ is 60. Find the common ratio.

55b. Sum of $0.33 + 0.33r + 0.33r^2 + 0.33r^3 + \ldots$ is 3. Find r (as a fraction).

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56a. The sum of the first 50 terms of an arithmetic sequence in which the common difference is $-3/2$ is 80. What is the first term?

56b. Sum of the first 99 terms of an arithmetic sequence in which d is $-5/3$ is 2000. Find the first term.

57) Use the binomial theorem and Pascal's Triangle to find $(x - y)^5$.

58) Find the 6th term of $(2x - 3y)^{12}$.

<u>Write each using sigma notation.</u>

59) 6 + 9 + 12 + 15 + 18 + 21

60) 2 + 6 + 18 + 54

61) 1 + 2 + 6 + 24 + 120

62) 3 + 8 + 15 + 24 + 35 + 48 + 63

<u>Simplify each.</u>

63) $\frac{32!}{30!}$

64) $\frac{100!}{99!}$

65) $\frac{(3n)!}{(3n-2)!}$

Study for Guided Notes for the Introduction to Calculus Unit. It is on the final but not part of this review!

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KEY

1) 1 2) and 3) proofs will vary 4) $\frac{2\sqrt{77}}{77}$ 5) 1/35 6) cos x 7) $\frac{2\sqrt{6}-2\sqrt{2}}{15}$

7a) ½ 7b) $3 + 4\cos(2x) + \cos(4x)$ 7c) $\sqrt{3}$

8) x = π/6 or 11π/6 9) x = 7π/6 or 11π/6 10) x = 63.4° or 243.4°

11) a) -24/25 b) -7/25 c) $\frac{\sqrt{5}}{5}$ d) 24/7 e) $\frac{-2\sqrt{5}}{5}$

12) $\frac{\sqrt{6}+\sqrt{2}}{4}$ 13) $\frac{-\sqrt{2+\sqrt{3}}}{2}$ you need to check your Quad $\frac{11\pi}{12}$ is in II & cosine is neg in II

14) x = 0, $\pi, \frac{\pi}{2}$ 15) $\frac{2\pi}{3}, \frac{4\pi}{3}, \pi$

16) x = 0, π, 1.25, 4.39 (for **all** values x = 0 + πk, 1.25 + πk)

17) x = π/6, 5π/6, 7π/6, 11π/6 (for **all** values, you can write x = π/6 + πk, 5π/6 + πk)

18) x = π/6 or 11π/6 19) x = 7π/6 or 11π/6 20) x = 63.4° or 243.4° 21) 0 + 2πk, $\frac{\pi}{2} + \pi$k 22) $\frac{\pi}{6} + \frac{2\pi}{3}$ k

23) $\frac{\pi}{4} + \frac{\pi}{3}$ k 24) $\frac{\pi}{2} + \frac{2\pi}{3}$ k , $\frac{3\pi}{2} + 2\pi$ 25) (3, −300°), (−3, 240°), (−3, −120°) 26) (10.77, 111.8°)

27) (2, −π/6), (−2, 5π/6), (2, 11π/6) 28) y = −3 29) r = −25 sec θ 30) limacon with no inner loop

31) 7-petal rose 32a) r = 3 − 5 sin θ 32b) $r = -7cos(5\theta)$ 32c) $r^2 = 16\sin(2\theta)$ 32d) $r = 4 + 2cos(\theta)$

33) 5.83(cos −30.96° + i sin −30.96°) 34) −62.1 − 231.8i

35) 14 cis 30° or 12.12 + 7i 36a) 75 cis 43° or 54.85 + 51.15i 36b) $2cis\left(\frac{\pi}{24}\right)$ $2cis\left(\frac{25\pi}{24}\right)$ $2cis\left(\frac{13\pi}{24}\right)$ $2cis\left(\frac{37\pi}{24}\right)$

37) −9i + 8j 38a) 33 38b) $\frac{3i\sqrt{13}}{13} - \frac{2j\sqrt{13}}{13}$ 39a) 35.9 39b) $4i\sqrt{3} - 4j$

40) 242.97 pounds at −10.76° 41) 355.4 mph at S4.73°E 42) 38.76 pounds, 38.15°

43a) A = 3, B = −2 43b) A = 5, B = 3, C = −2 43c) $\frac{2}{x-3} + \frac{2x+5}{x^2+3x+3}$ 44a) (2, 1) & (-2, - 1) 44b) (−1,0)& (1,0)

44c) (1, 2, 2) 45a) $\frac{(x+2)^2}{16} + \frac{(y-3)^2}{4} = 1$ 45b) $\frac{(x-4)^2}{1} + \frac{(y+1)^2}{16} = 1$ 46a) 14 46b) $4\sqrt{2}$ 47a) $4\sqrt{3}$

47b) $2\sqrt{61}$ 48a) $-8(y + 1) = (x - 2)^2$ 48b) $32(x - 3) = (y + 2)^2$ 49a) $(-2\sqrt{26}, 0)$ and $(2\sqrt{26}, 0)$

49b) $(0, 1-\sqrt{74})$ and $(0, 1+\sqrt{74})$ 50a) (−11, 0) and (11, 0) 50b) (3, 7) and (3, −11)

51a) $-16(x - 3) = (y - 2)^2$ 51b) $-180(y + 35) = (x - 220)^2$ 51c) $\frac{(y-5)^2}{4} - \frac{(x-3)^2}{25} = 1$ 52a) 268 52b) −394 53a) −814

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53b) $8939\frac{6}{11}$ 54a) 32 54b) 3/22 55a) 14/15 55b) 89/100 56a) 38.35 56b) $101\frac{86}{99}$

57) $x^5 - 5x^4y + 10x^3y^2 - 10x^2y^3 + 5xy^4 - y^5$ 58) $-24{,}634{,}368x^7y^5$ 9) $\sum_0^5 (3x+6)$ 60) $\sum_0^3 2(3)\hat{}x$

61) $\sum_1^5 x!$ 62) $\sum_2^8 x^2 - 1$ 63) 992 64) 100 65) (3n)(3n – 1)