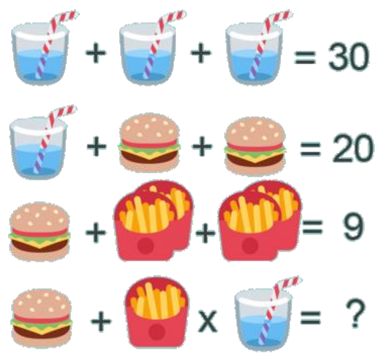
PreCalculus with TRIG – Unit 11 – Partial Fractions

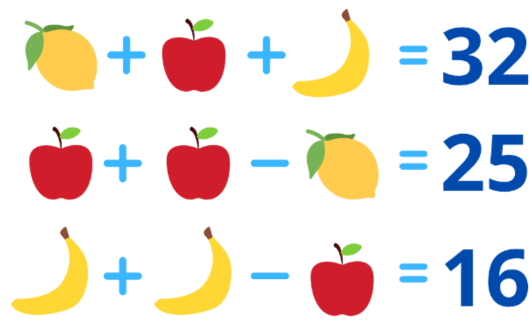
Day 1 – Section 7.2 – Solving Systems with 3 variables

Objectives: SWBAT Solve systems of **linear** equations with **3 variables.** Use elimination, substitution, or reduced row echelon form.

1) Solve the following:

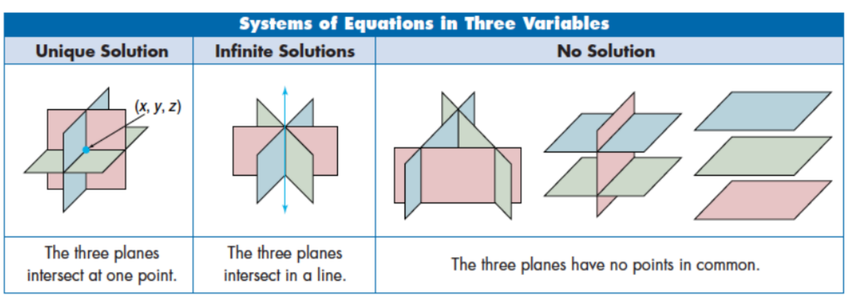


2) Solve the following:

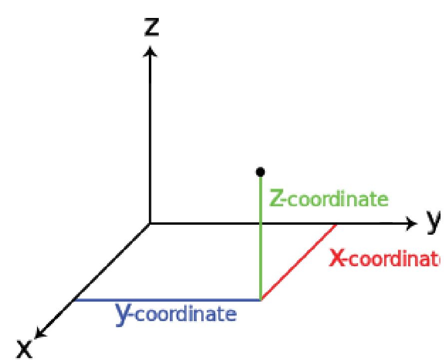


System of Equation with 3 Variables:

- A system of equations with three variables means you have three given equations. Your answer is an ordered triplet (x, y, z).
- This ordered triplet gives a true statement when plugged into each of the three equations.



Systems of Equations in Three Variables		
Unique Solution	Infinite Solutions	No Solution
The three planes intersect at one point.	The three planes intersect in a line.	The three planes have no points in common.



Is the following ordered triples solutions to the given systems?

1) $(1, -2, 1)$

$$2x + 3y + 2z = -2$$
$$x + y + z = 0$$
$$5x + y - z = 1$$

a) $(6, 0, -3)$

$$x + 4y - 2z = 12$$
$$3x - y + 4z = 6$$
$$-x + 3y + z = 9$$

Elimination Review of Algebra 1:

$$\begin{aligned} 2x - 7y &= -10 \\ 3x + 8y &= 22 \end{aligned}$$

In order to solve a system like this, you can use **elimination or substitution.**

THE ELIMINATION METHOD FOR A THREE-VARIABLE SYSTEM

1)

2)

3)

4)

5)

6)

7)

Solve the following systems by elimination.

2)
$$\begin{aligned} 2x + 3y - 2z &= 3 \\ x + y + z &= 3 \\ 4x + y - 3z &= 2 \end{aligned}$$

b)
$$\begin{aligned} 2x - y + 2z &= -7 \\ -x + 2y - 4z &= 5 \\ x + 4y - 6z &= -1 \end{aligned}$$

Using Matrices to Represent Systems:

Row Reduced Echelon Form

Solve by using a method involving matrices and RREF.

4) $\begin{aligned} 3x - y + 2z &= 7 \\ 6x - 2y + z &= 8 \\ 2x - y + 3z &= 8 \end{aligned}$

c) $\begin{aligned} x + y + 2z + 3w &= 7 \\ 2x - 5y + 4z - w &= 0 \end{aligned}$ $\quad$ $\begin{aligned} -x + y - 2z + 8w &= 6 \\ 4x - 5y + z + w &= 1 \end{aligned}$

5) If the following three points are on a parabola, write the equation that represents it in the form $y = ax^2 + bx + c$. $(-2,7)$, $(1,-2)$ and $(2,3)$ Use rref.

6) Last Tuesday, Hank Cinemas sold a total of 8500 movie tickets to watch *Fast and the Furious 21*. The sales totaled $64,600. Tickets can be bought in one of 3 ways: a matinee admission costs $5, student admission is $6 all day, and regular admissions are $8.50. How many of each type of ticket was sold if twice as many student tickets were sold as matinee tickets?

d) The Ready Player Huskie Arcade in Reno, NV uses 3 different colored tokens for their game machines. For $20 you can purchase any of the following mixtures of tokens: 14 gold, 20 silver, and 24 bronze; OR, 20 gold, 15 silver, and 19 bronze; OR, 30 gold, 5 silver, and 13 bronze. What is the monetary value of each token?

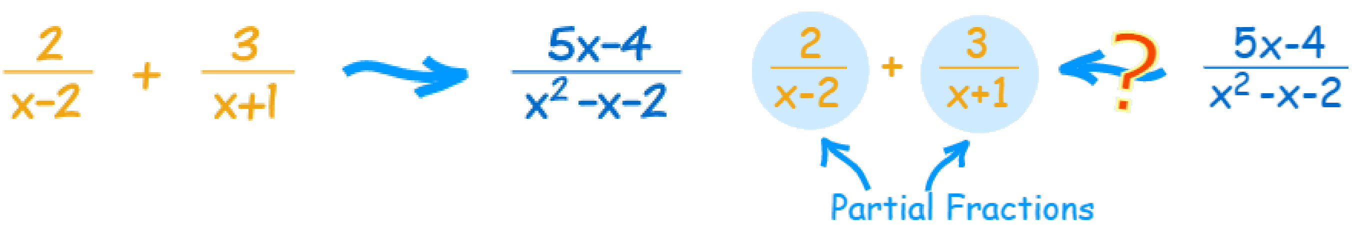
Day 2 – Section 7.3 – Introduction to Partial Fractions

Objectives: SWBAT Set up Partial fractions for Case #1 and Case #2.

Partial Fractions: Fractions that combine to yield another fraction

- When we add or subtract fractions, we get the same denominator and combine the numerators. The idea of finding partial fractions is reversing this thinking.

Simplifying Fractions: **Going Backwards AKA Partial Fractions**



Steps to Find Partial Fractions:

- **Identity the type(s) of Partial Fraction (I, II, III, and/or IV)**
- Set up the partial fractions with the unknown constants (A, B, C, etc.) in the numerators
- Multiply all terms so each term has a common denominator
- Use the Denominator Cancelation Rule to remove all denominators
- Simplify or Organize your x^2 terms, x terms, and constants
- Solve the linear system by using substitution or elimination or RREF

4 Types of Partial Fractions			
Type I	**Type II**	**Type III**	**Type IV**
• Denominator has no repeating factors • Is already factored or can be factored	Denominator has repeating factors that can be factored or already factored	• No repeated factors • Denominator has a prime quadratic factor (not factorable)	Denominator has repeating prime quadratic factors (not factorable)
$\dfrac{blah}{(x-1)(x+3)}$	$\dfrac{blah}{(x-1)^2}$	$\dfrac{blah}{(x+1)(x^2+6x-1)}$	$\dfrac{blah}{(x^2+1)^2}$
It is ALL About the Denominator			

Type I – Denominator has no repeated factors and is factorable/already factored

Find the partial fractions of the following.

1) $\frac{11x-10}{(x-2)(x+1)}$

2) $\frac{5x-4}{x^2-x-2}$

a) $\frac{x+14}{(x-4)(x+2)}$

Type II – Denominator has repeated factors and is factorable/already factored

3) $\frac{x-18}{x(x-3)^2}$

b) $\frac{x+2}{x(x-1)^2}$

Partial Fraction where you're A, B, or C are fractions/decimals:

4) $\frac{x}{x^2+2x-3}$

Day 3 – Section 7.3A – Partial Fractions – Part II

Objectives: SWBAT Set up Partial fractions for all 4 Cases

Review Questions of the day:

Deconstruct the fractions below

1) $\frac{7}{x^2+5x-6}$

2) $\frac{6x^2-x-36}{x^3-x^2-12x}$

Steps to Find Partial Fractions:

- Identity the type(s) of Partial Fraction (I, II, III, and/or IV)
- Set up the partial fractions with the unknown constants (A, B, C, etc.) in the numerators
- Multiply all terms so each term has a common denominator
- Use the Denominator Cancelation Rule to remove all denominators
- Simplify or Organize your x^2 terms, x terms, and constants
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It is All About the Denominator			

Type III – non-repeated, prime Quadratic

1) $\dfrac{8x^2+12x-20}{(x+3)(x^2+x+2)}$

a) $\dfrac{5x^2+6x+3}{(x+1)(x^2+2x+2)}$

Type IV – Repeated, Prime Quadratic

2) $\dfrac{5x^3-3x^2+7x-3}{(x^2+1)^2}$

b) $\dfrac{x^2+2x+3}{(x^2+4)^2}$

Identify the type of each fraction and rewrite it – just set up the partial fractions.

Expression	Type (how you know?)	Set up
$\dfrac{2x+1}{x^2+x+3}$		
$\dfrac{3x+2}{(x-1)(x+2)(x-3)}$		
$\dfrac{2x-3}{x^3-2x^2+x}$		
$\dfrac{x^2+2x+5}{(x-4)(x^2+3x-7)}$		
$\dfrac{4x^3+2x^2+1}{x^4+2x^2+1}$		

FOR NEXT CLASS NOTES: Please download the Desmos App onto your Cell Phone for graphing:

Desmos Graphing Calculator 4+
Desmos
#38 in Education
★★★★★ 4.2 • 928 Ratings
Free

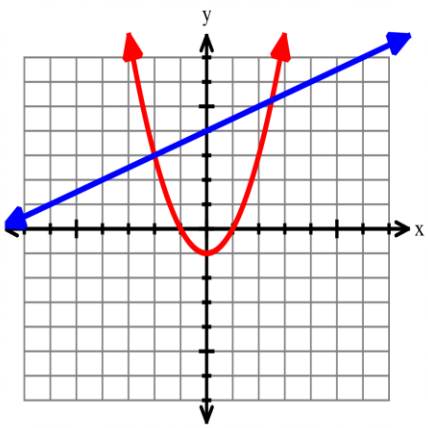
Day 4 – Section 7.4 – Solving Non-Linear Systems of Equations

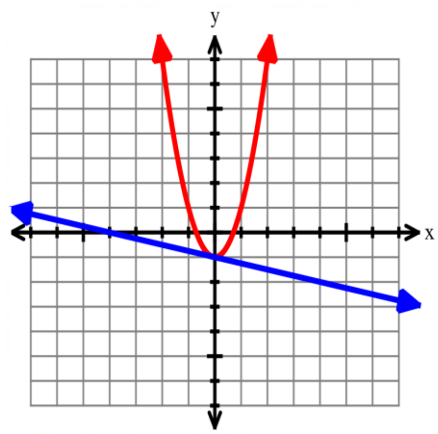
Objectives: SWBAT solve systems of equations

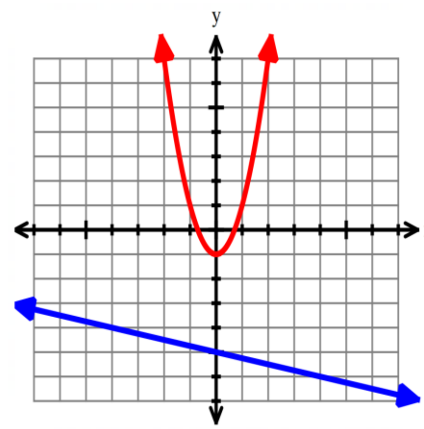
System of Equations:

Linear vs. Non-Linear:

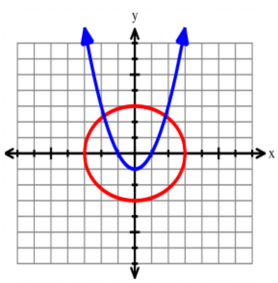
Number of Solutions for a Parabola and a Line:

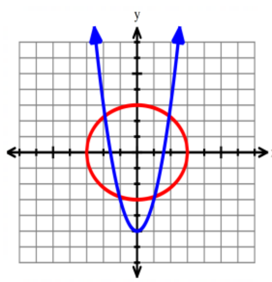


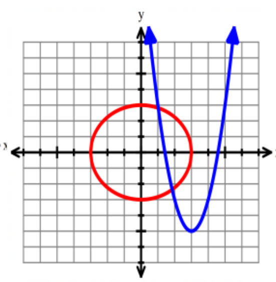


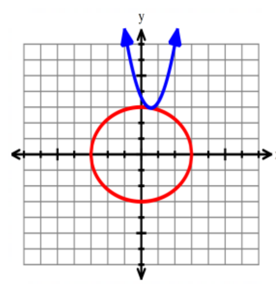


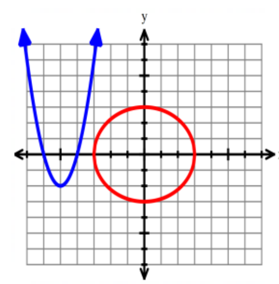
Number of Solutions for a Parabola and Circle:











Methods for Solving Systems of Equations:

Substitution:

Elimination:

Graphing:

When should I use each method?

Solve the following systems by Substitution:

1) $$\begin{aligned} xy &= 4 \\ x^2 + y^2 &= 8 \end{aligned}$$

2) $$\begin{aligned} 2x + 4y &= 4 \\ x^2 + 3x + 4y^2 - 3y &= 1 \end{aligned}$$

a) $$\begin{aligned} x + y &= -3 \\ x^2 + 2y^2 &= 12y + 18 \end{aligned}$$

Solve the following systems by Elimination:

3) $$\begin{aligned} 3x^2 - 2y^2 &= -5 \\ 2x^2 - y^2 &= -2 \end{aligned}$$

4) $$\begin{aligned} x^3 + y &= 0 \\ x^2 - y &= 0 \end{aligned}$$

Your Choice:

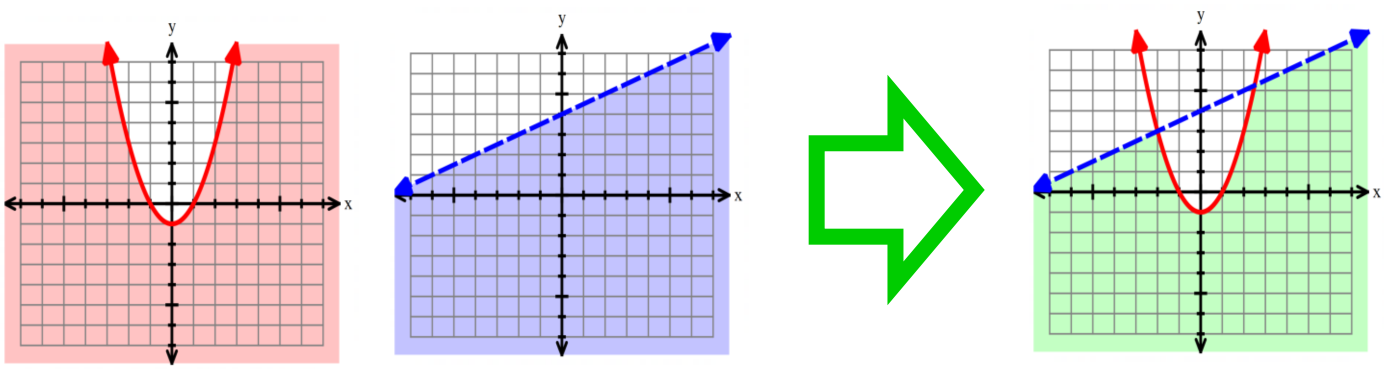
b) $$\begin{aligned} -4x + y &= 12 \\ y &= x^3 + 3x^2 \end{aligned}$$

Now go back, and solve each one with the Desmos Graphing Calculator App.

<u>Day 5 – Section 7.5 – Solving Systems of Inequalities</u>

Objectives: SWBAT solving systems of inequalities by graphing

<u>System of Inequalities:</u>

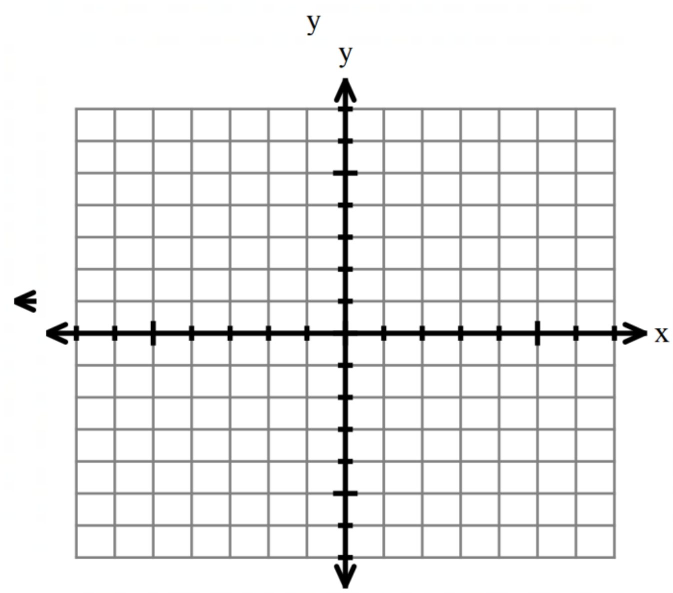


Important Reminders:

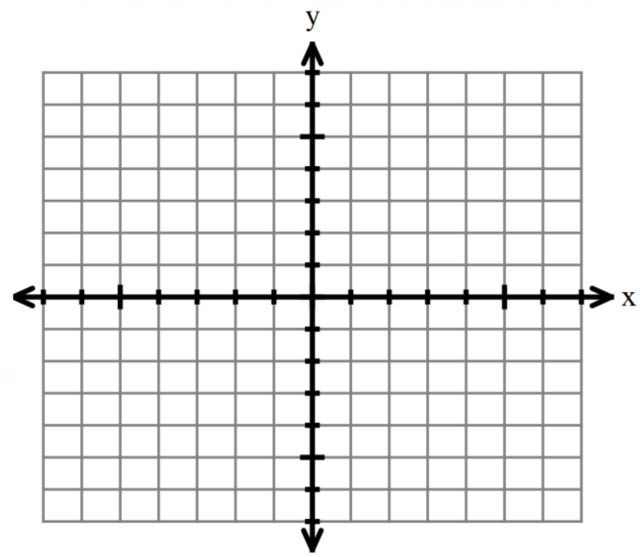
- **The answer to a system of inequalities is a ________________________ and is usually not a point.**
- **Your line or curve will be ________________ if there is a $\geq$ or $\leq$**
- **Your line or curve will be ________________ if there is a $>$ or $<$**
- **For each line or curve, you need to pick a point on either side of the line and curve the region that produces a _______________________ solution.**

<u>Graph the following system of inequalities.</u>

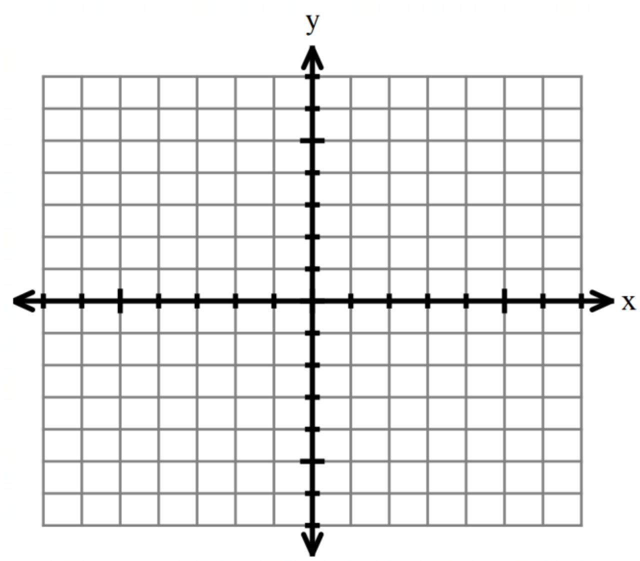
1) $x^2 + y^2 \leq 9$
$2x - y > 4$



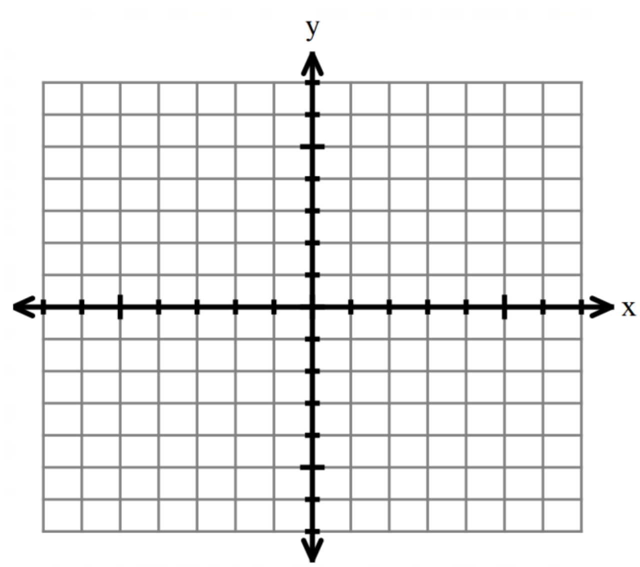
2)
$$\begin{aligned} 3x + y &\le 3 \\ x &> -1 \\ x &\le y^2 - 4 \end{aligned}$$



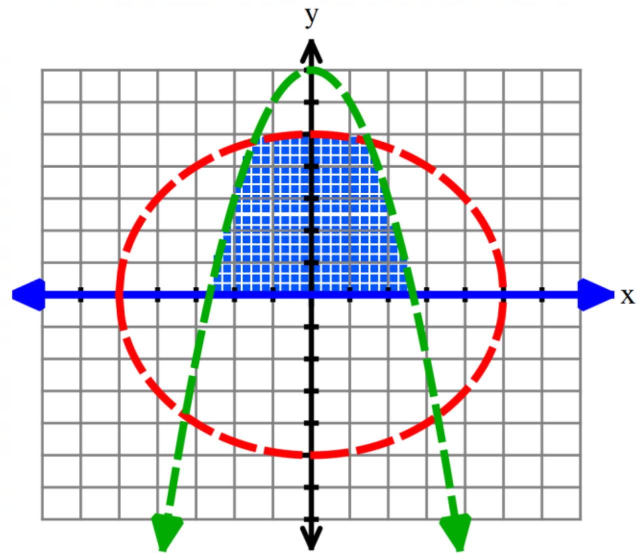
3)
$$\begin{aligned} y &< \frac{3}{x-2} + 3 \\ y &\ge 4x^2 - 1 \end{aligned}$$



a)
$$\begin{aligned} y &< 2^x \\ 4y^2 &\ge 36 - 4x^2 \end{aligned}$$



4) Write a system that represents the graphs below.

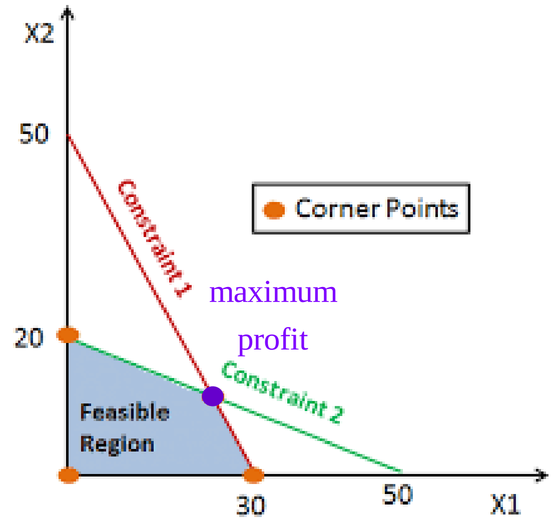


Day 6 – Section 7.6 – Introduction to Linear Programming

Objectives: SWBAT solve linear programming through graphing inequalities

Linear Programming:

- The process of maximizing or minimizing a function with certain constraints
- Examples: optimizing profit given constraints on labor expenses and cost of goods



Constraint:

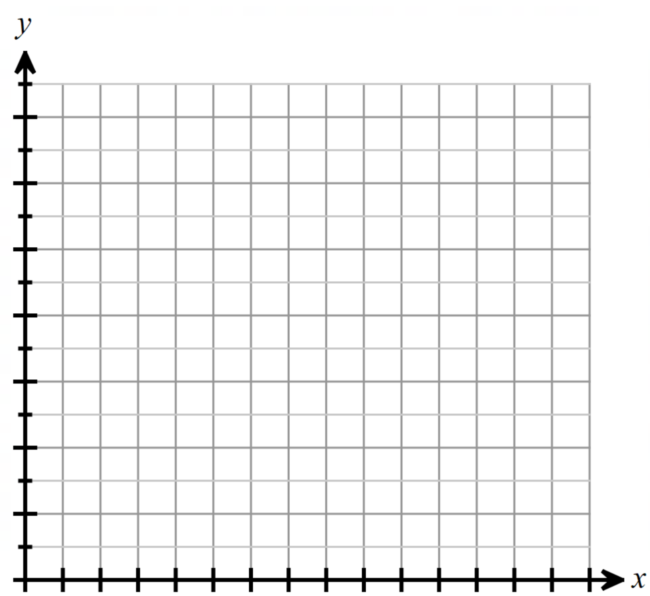
Steps for Success:

1. Graph the linear inequalities or constraints.
2. Shade the appropriate regions (it is not always under the graph).
3. Find the vertices (intersection points) of the polygon formed
4. Plug the vertices into the function you are trying to maximize or minimize
5. The highest y value is the ________________ and the lowest y value is the ________________.

1) Given the following constraints, and the objective function $f(x, y) = 3x + 4y$, use linear programing to maximize the function $f(x, y)$. Label your polygon's vertices as *z*.

$x + y \leq 6$ $\qquad$ $x \geq 0$

$2x + y \leq 14$ $\qquad$ $y \geq 0$



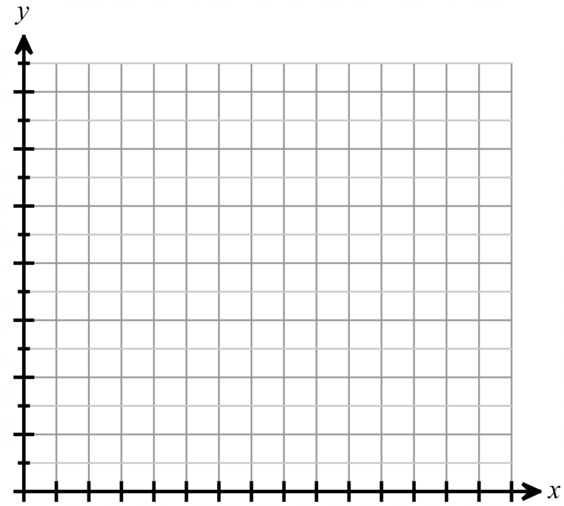
The vertices of the polygon formed are at:

____

The max is ____________ and occurs at _________.

2) Maximize the objective function, $z = 8x + 19.5y$, with the following constraints.

$x + y \geq 6 \qquad x \geq 0$
$2x + y \leq 12 \qquad y \geq 0$



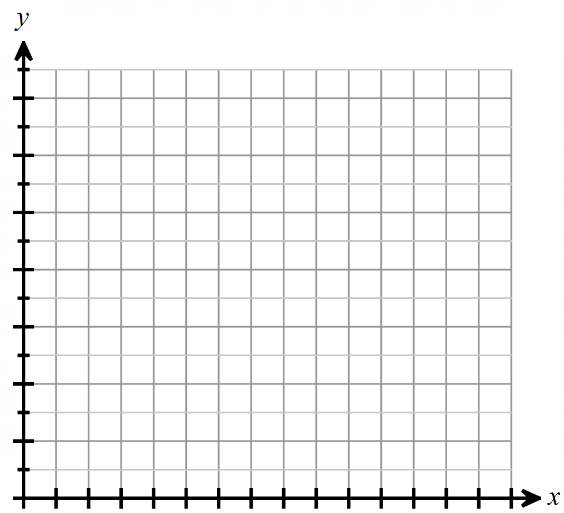
The vertices of the polygon formed are at:

____

The max is ____________ and occurs at _________.

a) Maximize the objective function, $z = 5x - \frac{y}{2}$, with the following constraints.

$3y - x \geq 6 \qquad x \leq 3$
$3y + 2x \leq 24 \qquad y \geq 0$



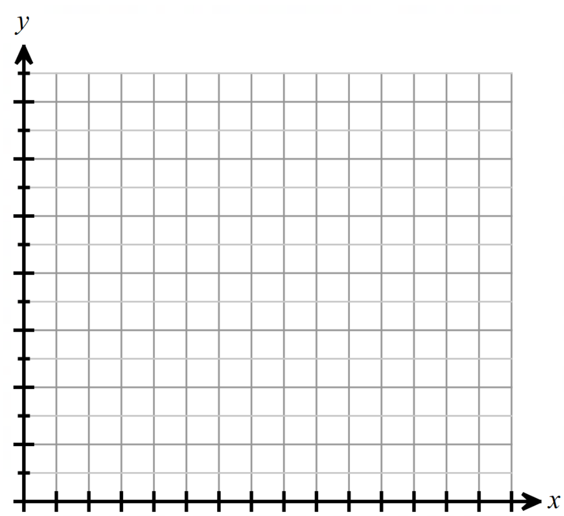
The vertices of the polygon formed are at:

____

The max is ____________ and occurs at _________.

3) A manufacturer produces two models of mountain bicycles. The times (in hours) required for assembling and paining each model are given by the following table.

	Model A	Model B
Assembly	5	4
Painting	2	3



The manufacture is subject to the following constraints:

- The assembly department can only work 200 hours a week
- The paint department can only 108 hours per week
- The profit for Model A is $27, and the profits for Model B is $18.

How many of each type should be produced to maximize profit?

<u>Day 6 – Section 7.6A – Writing Constraints for Linear Programming</u>

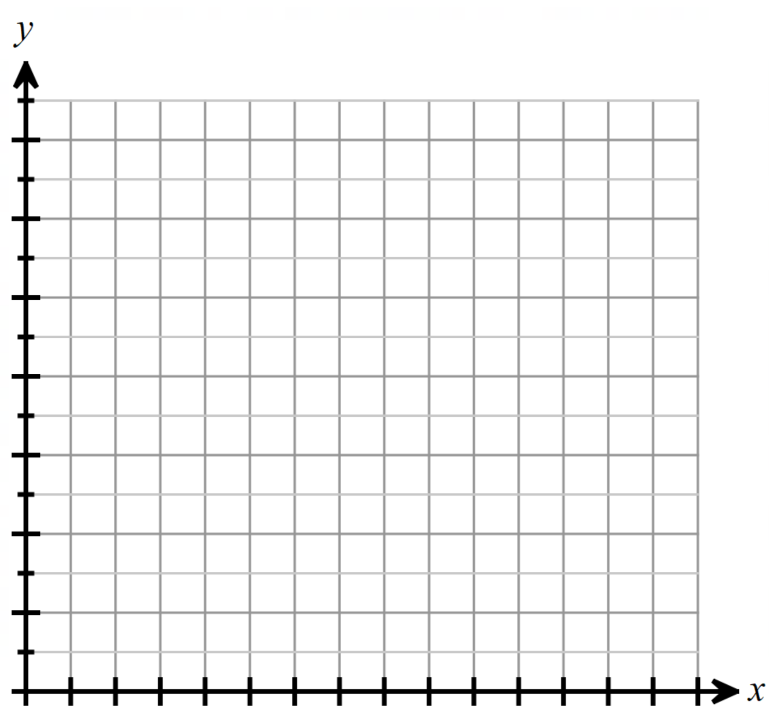
Objectives: SWBAT Write constraints and objective functions and maximize or minimize these functions through the use of linear programming

<u>Steps for Success:</u>

- **Decide what your axes are and label them accordingly (*x* or *y*)**
- **Decide what your max/min equation is really asking (write that equation with a *z*)**
- **Write 2 – 4 inequalities with the remaining pieces of information**
- **Can your examples include negatives values for either axis? If not, add $x \geq 0$ and $y \geq 0$ to your system of equations**
- **When graphing linear constraints, find the *x* and *y* intercepts**
- **Use a ruler / straight edge to graph your lines**

1) A Booth Street Players are presenting the show *Havoc the Huskie* for students and the community. Admission for parents is $8 and $4 for students. The theatre can hold only 128 occupants, and it is a requirement that every two parents must bring one student. Write a system of equations for this scenario only (do not solve)

2) You are about to take a Celebration of Knowledge that contains algebraic computation problems worth 6 points each and math modeling word problems that are 10 points each. You can do the algebra problems in 2 minutes each and the modeling problems in 4 minutes each. You have 40 minutes to take the test and may answer no more than 12 problems. If you answer every question correctly, how many of each type of problem must you answer to maximize your score? What is the max score?



3) On June 24, 1948, the former Soviet Union blocked all land and water routes through East Germany to Berlin. A gigantic airlift was organized using American and British planes to bring food, clothing, and other supplies to more than 2 million people in West Berlin. The cargo capacity was 30,000 cubic feet for an American plane, and 20,000 cubic feet for a British plane. To break the soviet blockade, the Western Allies had to maximize cargo capacity, but were subject to the follow restrictions.

- No more than 44 planes could be used.
- American Planes need personnel of 16 which was double the among of British planes.
- The total number of personnel could not exceed 512.
- The cost of an American flight was $9,000 and the cost of the British flight was $5,000.
- The total weekly costs could not exceed $300,000.

Find the number of American and British plans that were used to maximize cargo capacity.