PRECALCULUS GUIDED NOTES: Intro to Sequences and Series(10.1)

Name _____ Date _____

Review Questions

- 1) Solve for all values of x: $\cos^2 x + \cos x = 0$
- 2) Solve for all values of x. $sin(2x) = \frac{1}{2}$
- 3) Write parametric equations for the line y = 2x + 8.

Sequence: A list of values with a definite pattern

Series: A sum of values with a definite pattern

Σ NOTATION: We use Σ notation to represent a series. Σ stands for "find the sum".

Expand each series and find the sum.

Ex. 1
$$\sum_{\text{versus}} (2x+1)$$
 wersus $\sum_{\text{multiplied by a scalar (c)}} (2x+1)$

$$\sum_{Ex. 2}^{(4-5x)}$$

 $\sum x!$ Ex. 3

a) Simplify 8!/7!

b) Simplify 33!/32! C) Simplify (n + 1)!/n!

Ex. 4
$$\sum (3^x)$$

Ex. 5
$$\sum cosx$$

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Write each series using sigma or summation notation.

Ex.7 1+2+6+24+120

Ex. 8 1+4+9+16+25+36+49

Ex.9 2+4+6+8+10+12+14+16

Ex. 10 1 - 3 + 9 - 27 + 81

Ex. 11 $\sin 0 + \sin 1 + \sin 2 + \sin 3 + \sin 4$

Ex. 12 3 + 6 + 9 + 12 + ...

Ex. 13 2 + 5 + 10 + 17 + 26 + 37

Ex. 14 1+4+7+10+13+16+19

Ex. 15 -1 + 1 + 3 + 5 + 7 + 9 + 11 + ...

More Challenging Problems:

Ex. 16 (1) (4) + (2)(16) + (6)(64) + (24)(256)

Ex. 17 0+3+8+15+24+35+48 3 3 3 3 3 3 3 3 3

Ex. 18 1+4+6+4+1

Day 2Review Questions1) What is the formula for sin2x?

- 2) What is one formula for $\cos 2x$?
- 3) Solve for all values of $x \cos 5x = 1$.

Recursive Formula: A recursive formula for a sequence is used when you are describing the sequence in terms of a previous term in the sequence. Always write $a_1 = _$ and $a_n = _$.

A Perfect Example of a Recursive Formula

Fibonacci Sequence

1, 1, 2, 3, 5, 8, 13, 21, 34, etc

The following sequences are defined using recursion formulas. Write the first five terms of each sequence.

Ex. 1 $a_1 = 4$, $a_n = a_{n-1} + 3$ for $n \ge 2$

Ex. 2 $a_1 = 3$, $a_2 = 5(a_{n-1})$ for $n \ge 2$

Ex. 3 $a_1 = 4$, $a_2 = 5$, $a_n = a_{n-1} + a_{n-2}$ for n > 2

Ex. 4 $a_1 = 2$, $a_n = 3(a_{n-1}) + 3$ for $n \ge 2$

Ex. 5 $a_1 = 0$, $a_n = 2(a_{n-1})^2 + 1$ for $n \ge 2$

Factorial Practice

Expand each factorial and evaluate.

| Ex. 6 13!/12! | Ex. 7 20!/18! | Ex. 8 130!/129! |
|--------------------|----------------|--|
| Ex. 9 n!/(n-1)! | Ex. 10 50!/46! | Ex. 11 The number of arrangements in MISSISSIPPI |
| Ex. 12 $n!/(n-2)!$ | | Ex. 13 $(2n)!/(2n-2)!$ |

Write each series using summation notation. Use k for the index in each.

Ex. 14 $14^1 + 14^2 + 14^3 + \dots$ Ex. 15 4 + 7 + 10 + 13 + 16 + 19

Ex. 16
$$32 + 29 + 26 + 23 + 20 + 17$$

Ex. 17 $1 + 2 + 3 + 4 + ... + n$
 $1.2 + 1.2^2 + 1.2^3 + 1.2^4 + ... + n$

Ex. 18 $\tan 0 \sin 1 + \tan 1 \sin 2 + \tan 2 \sin 3 + \tan 3 \sin 4 + \tan 4 \sin 5 + \tan 5 \sin 6$

Ex. 19 2+7+14+23+34

Ex. 20 2+4+12+48+240

Precalculus Guided Notes: Arithmetic Sequences and Series 10.2

Day 3

Review Questions

- 1) How many triangles are possible? $A = 37^0$, b = 12, a = 7
- 2) When do you use Law of Cosines?
- 3) What is sin(x + y)?

Arithmetic Sequence: A list of numbers with a common difference (d) Arithmetic Series: A list of numbers with a common difference (d) written in summation form or with + signs in between

Finding the nth term of an arithmetic sequence: Consider the arithmetic sequence 3, 7, 11, 15, ...

What is the fifth term?

What is the 12th term?

What is the 112th term?

Put what you just did into a formula.

Formula for finding nth term of an arithmetic sequence

Ex. 1 Find the first five terms of an arithmetic sequence with $a_1 = 2$ and d = 4

Ex. 2 Find the first four terms of the sequence $a_1 = 3$, $a_n = a_{n-1} + 1.2$

Find the indicated term with the given a_1 and d.

Ex. 3 $a_{17} =$ ____, $a_1 = -2$, d = -4

Ex. 4 $a_{72} =$ ____, $a_1 = 23$, d = .5

Precalculus Guided Notes: Arithmetic Sequences and Series 10.2

Write a formula for the general term (nth term) of each arithmetic sequence. Then find the specified term.

Ex. 5 7, 10, 13, 16, ... Find a₈₉.

Ex. 6 9, 5, 1, -3, ... Find a₃₄.

Ex. 7 $a_1 = 24$, d = -7 Find a_{17} .

Ex. 8 $a_n = a_{n-1} + 2$, $a_1 = -17$ Find a_{28} .

Find the SUM of an arithmetic series: Consider the series 3 + 7 + 11 + 15 + 19 + 23

Using the information above, write the formula for an arithmetic sum.

FORMULA FOR SUM

SUBSTITUTE to the get second form

Ex. 9 Find the sum of the first 20 terms of 3 + 7 + 11 + ...

Ex. 10 Find the sum of the first 80 natural numbers 1 + 2 + 3 + ... + 80

Ex. 11 Find the sum of the first 50 positive even integers.

Ex. 12 Find the sum of the first 30 even integers.

Find each sum. Ex. 13 $\sum_{1}^{20} (2x+4)$ Ex. 14 $\sum_{0}^{50} (-3k+2)$ Ex. 15 $\sum_{1}^{101} (9k)$

Precalculus Guided Notes: Geometric Sequences and Series 10.3

Day 4
Review Questions
1) How many triangles are possible? a = 3, b = 7, c = 8
2) What is the formula for cos(x - y)?

Geometric Sequence is a list of numbers with a common ratio $r (r \neq 0)$. Geometric Series is a list of numbers with + signs in between, for which you are finding a sum.

r can always be found by dividing one term by its previous term $r = a_n/a_{n-1}$

Consider the sequence 3, 6, 12, 24, ...

What is the fifth term?

What is the 13th term?

What is the 113th term?

Put this into a formula.

Formula for finding nth term of a geometric sequence:

Ex. 1 Write the first four terms of a geometric sequence with $a_1 = 1.5$ and r = 2.

Ex. 2 Write the first four terms of a geometric sequence with $a_1 = 1$ and $a_n = a_{n-1}$ (-3).

Find each specified term by using the formula above

Ex. 3 $a_1 = 1.2$, r = -2 Find a_{17} .

Ex. 4 $a_1 = -2$, r = .5 Find a_{21} .

Precalculus Guided Notes: Geometric Sequences and Series 10.3

Write the general formula for each. Then find the specified term.

Ex. 5 4, 6, 9, ... Find a_{18} .

Ex. 6 3, -1, 1/3, ... Find a_{12} .

Sum of a GEOMETRIC SERIES

FORMULA is $S_n =$

Find the sum of each series.

Ex. 7 Find the sum of the first 21 terms of 2, 6, 18,...

Ex. 8
$$\sum_{1}^{7} (4)^{x}$$

Infinite Geometric Series: A series with an infinite number of values. The sum of this can be found when -1 < r < 1 ($r \neq 0$) because the values will be approaching a certain number.

Consider the series 3 + 1 + 1/3 + 1/9 + ...

Find the sum of this series. What should the sum be close to?

FORMULA FOR INFINITE SERIES S_{∞} =

Ex. 9 Find the sum of $4 + 2 + 1 + \frac{1}{2} + ...$

Use the sum formula to find the fractional form of each repeating decimal.

| Ex. 13 .89 Ex. 14 | 2.45 Ex. 15 .17 |
|-------------------|-----------------|
|-------------------|-----------------|

Day 5

Review Questions

- 1) What is sin2x?
- 2) What is tan0?
- 3) What is sec0?

Expanding a binomial means you must do FOIL at least one time. However, when the power is 3 or higher, this may be quite tedious... $(x+y)^3 (x+y)^4 (x+y)^5$ etc. On these, it would be easier to use the binomial theorem.

Ex. 1 $(x + y)^4$

Binomial Theorem:

(x + y)ⁿ =

For coefficients,

Use Pascal's Triangle:

Write with choose ${}_{n}C_{r} = {n \choose r}$

Ex.2 $(x - y)^5$

Ex. 3 $(2x + y)^6$

Ex. 4 (3x – 2y)⁵

Find the designated term for each binomial expansion.

Ex. 5 5^{th} term of $(x + y)^{12}$

Ex. 6 9^{th} term of $(w - y)^{15}$

Ex. 7 7^{th} term of $(2a - 3b)^{12}$

Ex. 8 12^{th} term of $(3y - x)^{16}$