Homefun Answers

3.1	**Exponential Functions**	**https://youtu.be/ZCrEs48DSWM**	**Pgs. 448-449: 11, 17, 19-24, 25-33 odd, 39, 41, 43, 52, 53, 55**

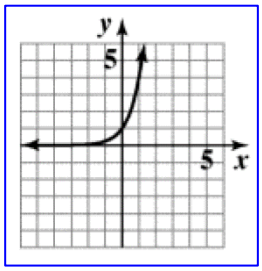
In Exercises 11–18, graph each function by making a table of coordinates. If applicable, use a graphing utility to confirm your hand-drawn graph.

11. $f(x) = 4^x$

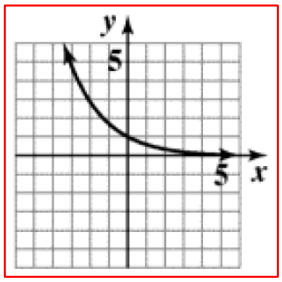
12. $f(x) = 5^x$

17. $f(x) = (0.6)^x$

#11



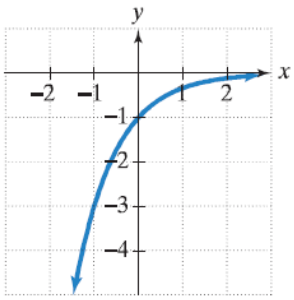
#17



In Exercises 19–24, the graph of an exponential function is given. Select the function for each graph from the following options:

$$f(x) = 3^x,\ g(x) = 3^{x-1},\ h(x) = 3^x - 1,$$
$$F(x) = -3^x,\ G(x) = 3^{-x},\ H(x) = -3^{-x}.$$

19.



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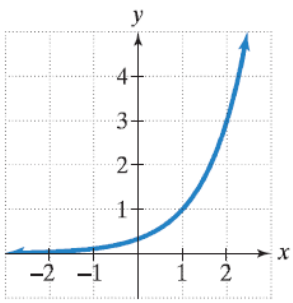

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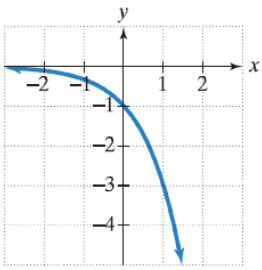

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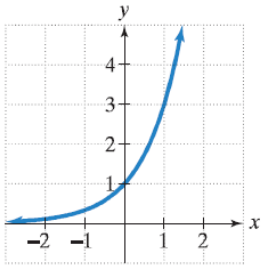

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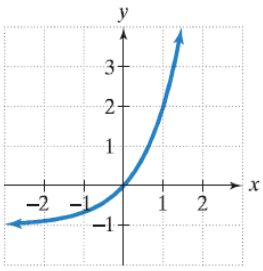

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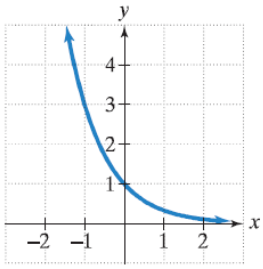


#19

$H(x) = -3^{-x}$

#20

$g(x) = 3^{x-4}$

#21

$g(x) = -3^x$

#22

$F(x) = 3^x$

#23

$F(x) = 3^x - 1$

#24

$F(x) = 3^{-x}$

In Exercises 25–34, begin by graphing $f(x) = 2^x$. Then use transformations of this graph to graph the given function. Be sure to graph and give equations of the asymptotes. Use the graphs to determine each function's domain and range. If applicable, use a graphing utility to confirm your hand-drawn graphs.

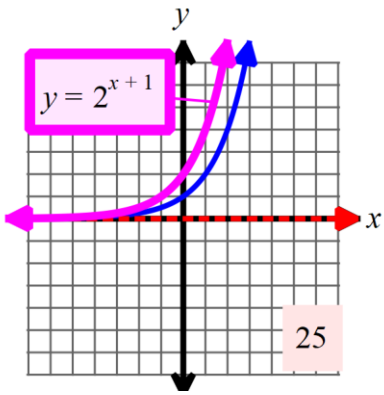
25. $g(x) = 2^{x+1}$

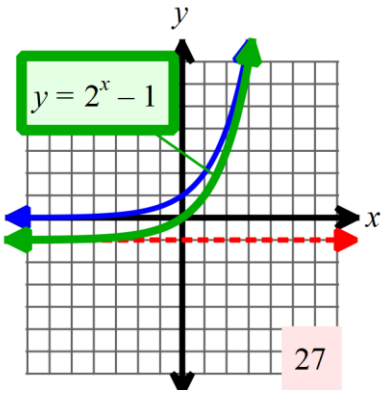
27. $g(x) = 2^x - 1$

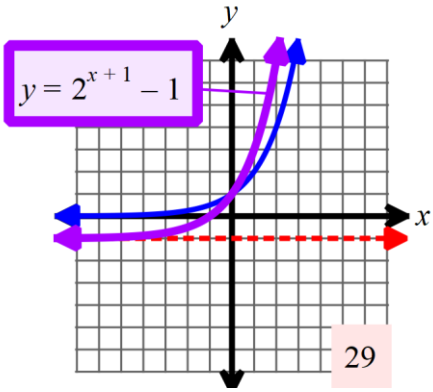
29. $h(x) = 2^{x+1} - 1$

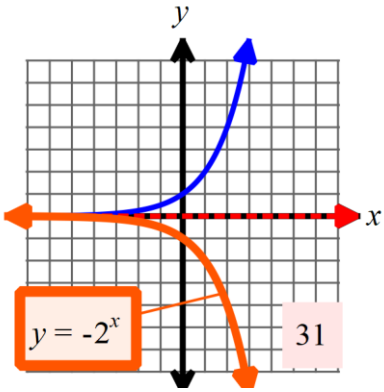
31. $g(x) = -2^x$

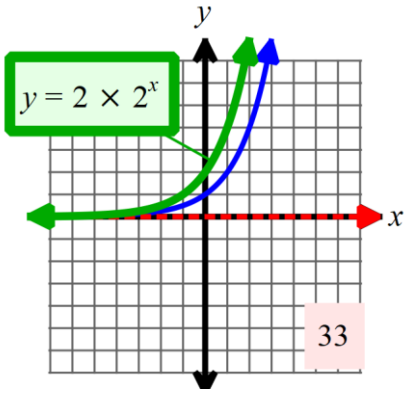
33. $g(x) = 2 \cdot 2^x$



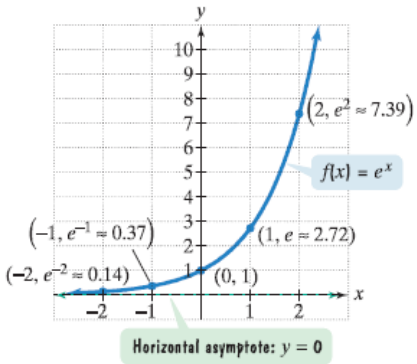








The figure shows the graph of $f(x) = e^x$. In Exercises 35–46, use transformations of this graph to graph each function. Be sure to give equations of the asymptotes. Use the graphs to determine each function's domain and range. If applicable, use a graphing utility to confirm your hand-drawn graphs.



35. $g(x) = e^{x-1}$

36. $g(x) = e^{x+1}$

37. $g(x) = e^x + 2$

38. $g(x) = e^x - 1$

39. $h(x) = e^{x-1} + 2$

40. $h(x) = e^{x+1} - 1$

41. $h(x) = e^{-x}$

42. $h(x) = -e^x$

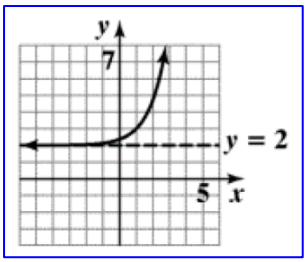
43. $g(x) = 2e^x$

44. $g(x) = \frac{1}{2}e^x$

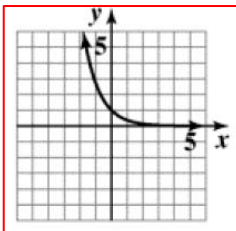
45. $h(x) = e^{2x} + 1$

46. $h(x) = e^{\frac{x}{2}} + 2$

#39



#41

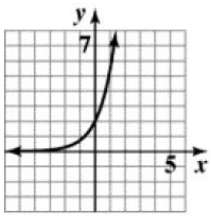


$h(x) = e^{-x}$

asymptote: $y = 0$

domain: $(-\infty, \infty)$; range: $(0, \infty)$

#43



$g(x) = 2e^x$

asymptote: $y = 0$

domain: $(-\infty, \infty)$; range: $(0, \infty)$

In Exercises 47–52, graph functions f and g in the same rectangular coordinate system. Graph and give equations of all asymptotes. If applicable, use a graphing utility to confirm your hand-drawn graphs.

47. $f(x) = 3^x$ and $g(x) = 3^{-x}$

48. $f(x) = 3^x$ and $g(x) = -3^x$

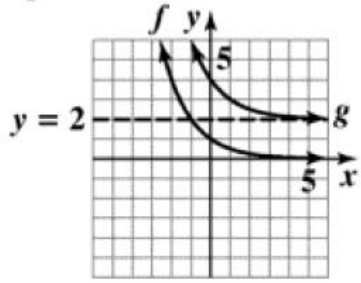
49. $f(x) = 3^x$ and $g(x) = \frac{1}{3} \cdot 3^x$

50. $f(x) = 3^x$ and $g(x) = 3 \cdot 3^x$

51. $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{2}\right)^{x-1} + 1$

52. $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{2}\right)^{x-1} + 2$

#52



$f(x) = \left(\frac{1}{2}\right)^x$

$g(x) = \left(\frac{1}{2}\right)^{x-1} + 2$

asymptote of $f(x)$: $y = 0$

asymptote of $g(x)$: $y = 2$

Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ *and* $A = Pe^{rt}$ *to solve Exercises 53–56. Round answers to the nearest cent.*

53. Find the accumulated value of an investment of \$10,000 for 5 years at an interest rate of 5.5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly; **d.** compounded continuously.

54. Find the accumulated value of an investment of \$5000 for 10 years at an interest rate of 6.5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly; **d.** compounded continuously.

55. Suppose that you have \$12,000 to invest. Which investment yields the greater return over 3 years: 7% compounded monthly or 6.85% compounded continuously?

#53

A. \$13,116.51

B. \$13,140.67

C. \$13,157.04

D. \$13,165.31

#55

7%

3.2	Introduction to Logarithms	https://youtu.be/HC1IXS7UW_w	Pgs. 463 – 465: 5-15 odd, 21-42, 81-99 odd

In Exercises 1–8, write each equation in its equivalent exponential form.

1. $4 = \log_2 16$ **2.** $6 = \log_2 64$

3. $2 = \log_3 x$ **4.** $2 = \log_9 x$

5. $5 = \log_b 32$ **6.** $3 = \log_b 27$

7. $\log_6 216 = y$ **8.** $\log_5 125 = y$

#5

32

#7

$6^y = 216$

In Exercises 9–20, write each equation in its equivalent logarithmic form.

9. $2^3 = 8$ **10.** $5^4 = 625$ **11.** $2^{-4} = \frac{1}{16}$ **12.** $5^{-3} = \frac{1}{125}$

13. $\sqrt[3]{8} = 2$ **14.** $\sqrt[3]{64} = 4$ **15.** $13^2 = x$ **16.** $15^2 = x$

17. $b^3 = 1000$ **18.** $b^3 = 343$ **19.** $7^y = 200$ **20.** $8^y = 300$

#9

$\log_2 8 = 3$

#11

$\log_2 \frac{1}{16} = -4$

#13

$\log_2 8 = \frac{1}{3}$

#15

$\log_{13} x = 2$

In Exercises 21–42, evaluate each expression without using a calculator.

21. $\log_4 16$ 2 **22.** $\log_7 49$ 2 **23.** $\log_2 64$ 6 **24.** $\log_3 27$ 3

25. $\log_5 \frac{1}{5}$ -1 **26.** $\log_6 \frac{1}{6}$ -1 **27.** $\log_2 \frac{1}{8}$ -3 **28.** $\log_3 \frac{1}{9}$ -2

29. $\log_7 \sqrt{7}$ $\frac{1}{2}$ **30.** $\log_6 \sqrt{6}$ $\frac{1}{2}$ **31.** $\log_2 \frac{1}{\sqrt{2}}$ $-\frac{1}{2}$ **32.** $\log_3 \frac{1}{\sqrt{3}}$ $-\frac{1}{2}$

33. $\log_{64} 8$ $\frac{1}{2}$ **34.** $\log_{81} 9$ $\frac{1}{2}$ **35.** $\log_5 5$ 1 **36.** $\log_{11} 11$ 1

37. $\log_4 1$ 0 **38.** $\log_6 1$ 0 **39.** $\log_5 5^7$ 7 **40.** $\log_4 4^6$ 6

41. $8^{\log_8 19}$ 19 **42.** $7^{\log_7 23}$ 26

In Exercises 81–100, evaluate or simplify each expression without using a calculator.

81. $\log 100$ 2 **82.** $\log 1000$ **83.** $\log 10^7$ 7 **84.** $\log 10^8$

85. $10^{\log 33}$ 33 **86.** $10^{\log 53}$ **87.** $\ln 1$ 0 **88.** $\ln e$

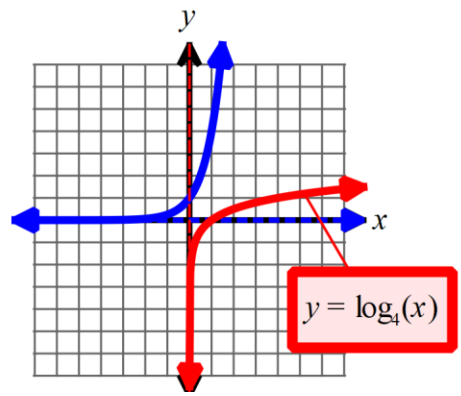
89. $\ln e^6$ 6 **90.** $\ln e^7$ **91.** $\ln \frac{1}{e^6}$ -6 **92.** $\ln \frac{1}{e^7}$

93. $e^{\ln 125}$ 125 **94.** $e^{\ln 300}$ **95.** $\ln e^{9x}$ $9x$ **96.** $\ln e^{13x}$

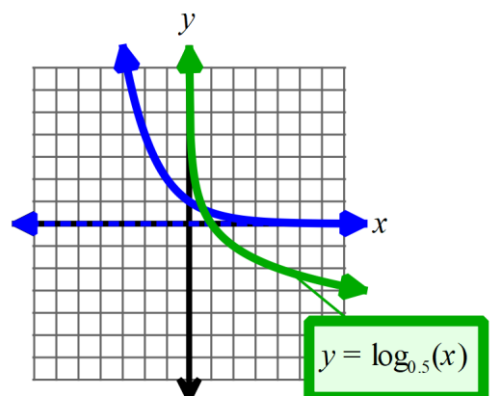
97. $e^{\ln 5x^2}$ $5x^2$ **98.** $e^{\ln 7x^2}$ **99.** $10^{\log \sqrt{x}}$ $\sqrt{x}$ **100.** $10^{\log \sqrt[3]{x}}$

3.2A	**Graphs of Logarithms**	**https://youtu.be/gA5-Vpo0OwY**	**Pgs. 463 – 465: 43, 45, 47-52, 53-71 odd, 75-78**

43. Graph $f(x) = 4^x$ and $g(x) = \log_4 x$ in the same rectangular coordinate system.



45. Graph $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{\frac{1}{2}} x$ in the same rectangular coordinate system.

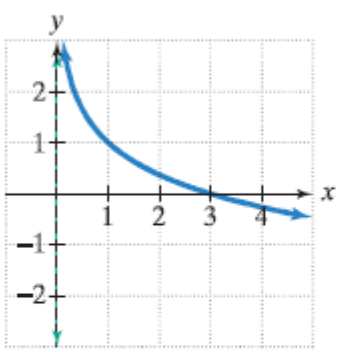


In Exercises 47–52, the graph of a logarithmic function is given. Select the function for each graph from the following options:

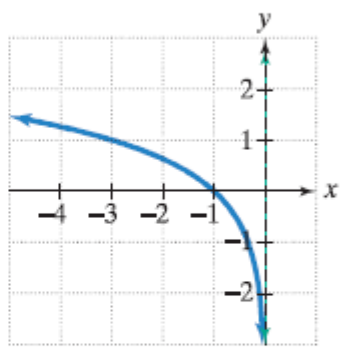
$$f(x) = \log_3 x,\ g(x) = \log_3(x-1),\ h(x) = \log_3 x - 1,$$

$$F(x) = -\log_3 x,\ G(x) = \log_3(-x),\ H(x) = 1 - \log_3 x.$$

47.



48.



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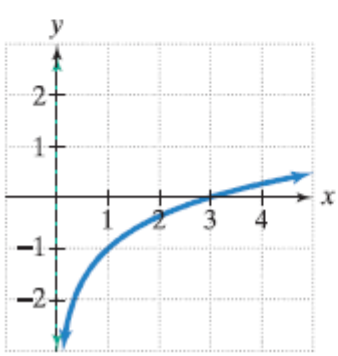
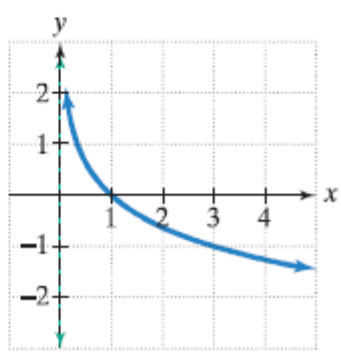


50.



#47

$H(x) = -\log_3(x) + 1$

#48

$G(x) = \log_3(-x)$

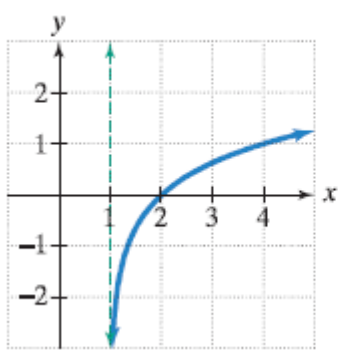
#49

$h(x) = \log_3(x) - 1$

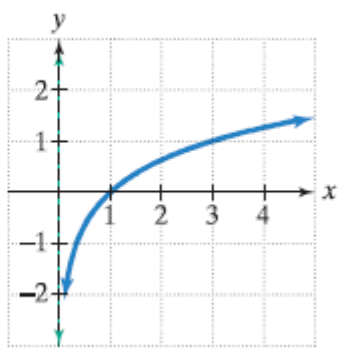
#50

$F(x) = -\log_3(x)$

51.



52.



#51

$h(x) = \log_3(x-1)$

#52

$f(x) = \log_3(x)$

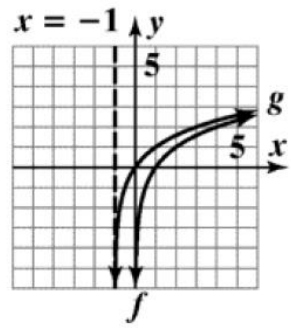
In Exercises 53–58, begin by graphing $f(x) = \log_2 x$. Then use transformations of this graph to graph the given function. What is the vertical asymptote? Use the graphs to determine each function's domain and range.

53. $g(x) = \log_2(x + 1)$

55. $h(x) = 1 + \log_2 x$

57. $g(x) = \frac{1}{2}\log_2 x$

53.



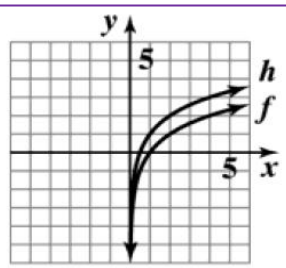
$f(x) = \log_2 x$

$g(x) = \log_2(x+1)$

vertical asymptote: $x = -1$

domain: $(-1, \infty)$; range: $(-\infty, \infty)$

55.



$f(x) = \log_2 x$

$h(x) = 1 + \log_2 x$

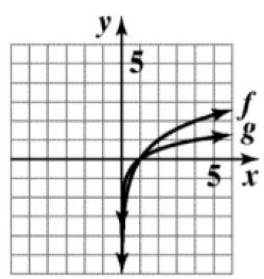
vertical asymptote: $x = 0$

$h(x) = 1 + \log_2 x$

vertical asymptote: $x = 0$

domain: $(0, \infty)$; range: $(-\infty, \infty)$

57.



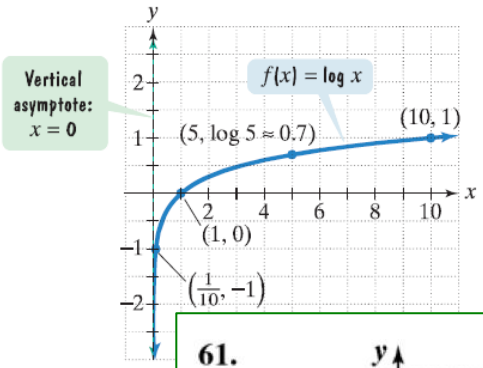
$f(x) = \log_2 x$

$g(x) = \frac{1}{2}\log_2 x$

vertical asymptote: $x = 0$

domain: $(0, \infty)$; range: $(-\infty, \infty)$

The figure shows the graph of $f(x) = \log x$. In Exercises 59–64, use transformations of this graph to graph each function. Graph and give equations of the asymptotes. Use the graphs to determine each function's domain and range.

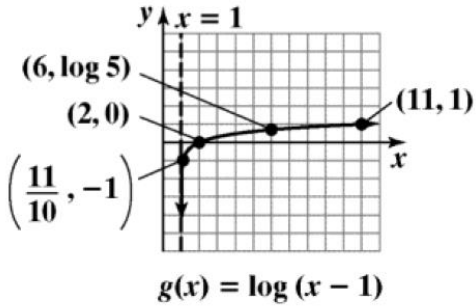


59. $g(x) = \log(x - 1)$

61. $h(x) = \log x - 1$

63. $g(x) = 1 - \log x$

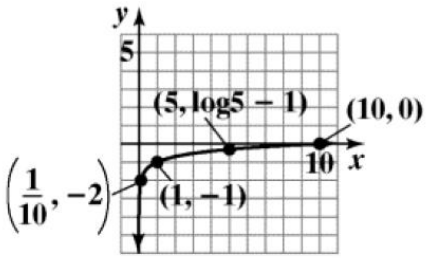
59.



$g(x) = \log(x - 1)$

vertical asymptote: $x = 1$
domain: $(1, \infty)$; range: $(-\infty, \infty)$

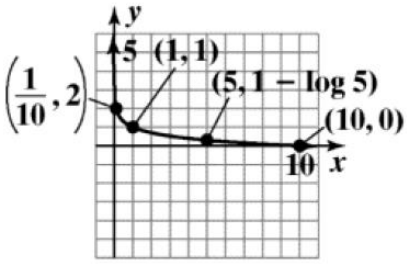
61.



$h(x) = \log x - 1$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

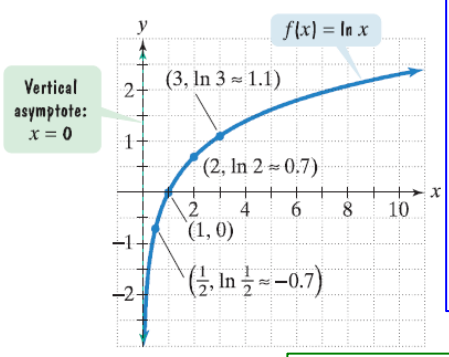
63.



$g(x) = 1 - \log x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

The figure shows the graph of $f(x) = \ln x$. In Exercises 65–74, use transformations of this graph to graph each function. Graph and give equations of the asymptotes. Use the graphs to determine each function's domain and range.



65. $g(x) = \ln(x + 2)$

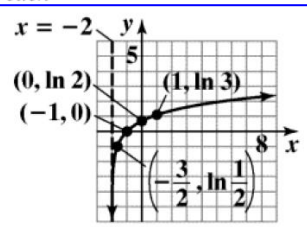
67. $h(x) = \ln(2x)$

69. $g(x) = 2 \ln x$

71. $h(x) = -\ln x$

73. $g(x) = 2 - \ln x$

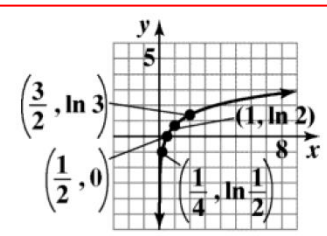
65.



$g(x) = \ln(x + 2)$

vertical asymptote: $x = -2$
domain: $(-2, \infty)$; range: $(-\infty, \infty)$

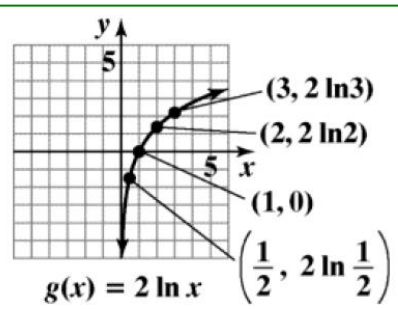
67.



$h(x) = \ln(2x)$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

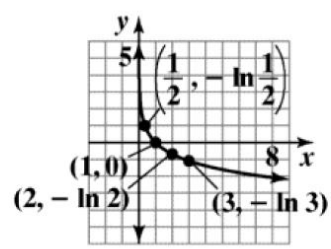
69.



$g(x) = 2 \ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

71.



$h(x) = -\ln x$

vertical asymptote: $x = 0$
domain: $(0, \infty)$; range: $(-\infty, \infty)$

In Exercises 75–80, find the domain of each logarithmic function.

75. $f(x) = \log_5(x + 4)$

76. $f(x) = \log_5(x + 6)$

77. $f(x) = \log(2 - x)$

78. $f(x) = \log(7 - x)$

#75	#76	#77	#78
$(-4, \infty)$	$(-6, \infty)$	$(-\infty, 2)$	$(-\infty, 7)$

3.2B	Logs, Natural Logs, Domain and Range	https://youtu.be/QLAFI19-I1k	3.2B Worksheet

Find the Domain of each function. Write your answers using interval notation.

1. $\log(3-x)$

$(-\infty, 3)$

2. $\log(2-7x)$

$\left(-\infty, \frac{2}{7}\right)$

3. $\ln(x^2-x-2)$

$(-\infty,-1)U(2,\infty)$

4. $\ln(x^2-4x-12)$

$(-\infty,-2)U(6,\infty)$

5. $\log\left(\frac{x+1}{x-5}\right)$

$(-\infty,-1)U(5,\infty)$

6. $\log(x^3-2x^2+x)$

$(0,1)U(1,\infty)$

Find the inverse of each function.

7. $f(x)=\log(x+2)-4$

$f^{-1}(x)=10^{x+4}-2$

8. $g(x)=-\ln(-x-2)+4$

$g^{-1}(x)=-e^{-x+4}-2$

9. $f(x)=log_3(x-3)-5$

$f^{-1}(x)=3^{x+5}+3$

10. $g(x)=log_7(-x)-1$

$g^{-1}(x)=-7^{x+1}$

11. $y=e^{x+2}-1$

$y^{-1}=\ln(x+1)-2$

12. $h(x)=-7^x+2$

$h^{-1}(x)=\log_7(-x+2)$

13. $f(x)=8^{x+7}-11$

$f^{-1}(x)=\log_8(x+11)-7$

14. $g(x)=4^{-x+2}-14$

$g^{-1}(x)=-\log_4(x+14)+2$

15. Find the domain of

$y=\ln(x^2-24)+1$. Leave your answer in reduced radical form.

$\left(-\infty,-2\sqrt{6}\right)U\left(2\sqrt{6},\infty\right)$

3.3	**Properties of Logs and Change of Base**	**https://youtu.be/pslSN2qLH1g**	**Pg. 475 – 477: 5-15 odd, 21-33 odd, 37, 44-62 even, 70-73, 81**

5. $\log(1000x) = \log 1000 + \log x = 3 + \log x$

7. $\log_7\left(\frac{7}{x}\right) = \log_7 7 - \log_7 x = 1 - \log_7 x$

9. $\log\left(\frac{x}{100}\right) = \log x - \log 100 = \log x - 2$

11. $\log_4\left(\frac{64}{y}\right) = \log_4 64 - \log_4 y$

$= 3 - \log_4 y$

13. $\ln\left(\frac{e^2}{5}\right) = \ln e^2 - \ln 5 = 2\ln e - \ln 5 = 2 - \ln 5$

15. $\log_b x^3 = 3\log_b x$

21. $\log_b x^2 y = \log_b x^2 + \log_b y = 2\log_b x + \log_b y$

23. $\log_4\left(\frac{\sqrt{x}}{64}\right) = \log_4 x^{1/2} - \log_4 64 = \frac{1}{2}\log_4 x - 3$

25. $\log_6\left(\frac{36}{\sqrt{x+1}}\right) = \log_6 36 - \log_6 (x+1)^{1/2}$

$= 2 - \frac{1}{2}\log_6 (x+1)$

27. $\log_b\left(\frac{x^2 y}{z^2}\right) = \log_b\left(x^2 y\right) - \log_b z^2$

$= \log_b x^2 + \log_b y - \log_b z^2$

$= 2\log_b x + \log_b y - 2\log_b z$

29. $\log\sqrt{100x} = \log(100x)^{1/2}$

$= \frac{1}{2}\log(100x)$

$= \frac{1}{2}(\log 100 + \log x)$

$= \frac{1}{2}(2 + \log x)$

$= 1 + \frac{1}{2}\log x$

31. $\log\sqrt[3]{\frac{x}{y}} = \log\left(\frac{x}{y}\right)^{1/3}$

$= \frac{1}{3}\left[\log\left(\frac{x}{y}\right)\right]$

$= \frac{1}{3}(\log x - \log y)$

$= \frac{1}{3}\log x - \frac{1}{3}\log y$

33. $\log_b \dfrac{\sqrt{x}\,y^3}{z^3}$

$= \log_b x^{1/2} + \log_b y^3 - \log_b z^3$

$= \dfrac{1}{2}\log_b x + 3\log_b y - 3\log_b z$

37. $\ln\left[\dfrac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$

$= \ln x^3 + \ln\sqrt{x^2+1} - \ln(x+1)^4$

$= 3\ln x + \dfrac{1}{2}\ln(x^2+1) - 4\ln(x+1)$

44. $\ln x + \ln 3 = \ln(3x)$

46. $\log_3 405 - \log_3 5 = \log_3\left(\dfrac{405}{5}\right)$

$= \log_3 81$

$= 4$

48. $\log(3x+7) - \log x = \log\left(\dfrac{3x+7}{x}\right)$

50. $\log x + 7\log y = \log x + \log y^7 = \log(xy^7)$

52. $\dfrac{1}{3}\ln x + \ln y = \ln x^{1/3} + \ln y$

$= \ln\left(x^{1/3}y\right)$ or $\ln\left(y\sqrt[3]{x}\right)$

54. $5\log_b x + 6\log_b y = \log_b x^5 + \log_b y^6$

$= \log_b\left(x^5y^6\right)$

56. $7\ln x - 3\ln y = \ln x^7 - \ln y^3 = \ln\left(\frac{x^7}{y^3}\right)$

58. $2\ln x - \frac{1}{2}\ln y = \ln x^2 - \ln y^{1/2}$

$= \ln\left(\frac{x^2}{y^{1/2}}\right)$ or $\ln\left(\frac{x^2}{\sqrt{y}}\right)$

60. $8\ln(x+9) - 4\ln x = \ln(x+9)^8 - \ln x^4$

$= \ln\frac{(x+9)^8}{x^4}$

62. $4\ln x + 7\ln y - 3\ln z$

$= \ln x^4 + \ln y^7 - \ln z^3$

$= \ln\frac{x^4 y^7}{z^3}$

70. $\log x + \log\left(x^2-4\right) - \log 15 - \log(x+2)$

$= \log x + \log\left(x^2-4\right) - \left(\log 15 + \log(x+2)\right)$

$= \log\left(x\left(x^2-4\right)\right) - \log\left(15(x+2)\right)$

$= \log\frac{x\left(x^2-4\right)}{15(x+2)}$

$= \log\frac{x(x+2)(x-2)}{15(x+2)}$

72. $\log_6 17 = \frac{\log 17}{\log 6} \approx 1.5812$

73. $\log_{14} 87.5 = \frac{\ln 87.5}{\ln 14} \approx 1.6944$

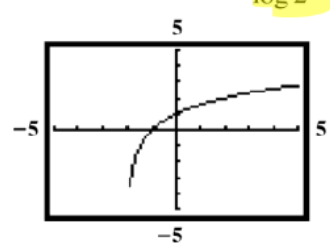
81. $y = \log_2(x+2) = \frac{\log(x+2)}{\log 2}$

$= \log\frac{x(x+2)(x-2)}{15(x+2)}$

$= \log\frac{x(x-2)}{15}$

71. $\log_5 13 = \frac{\log 13}{\log 5} \approx 1.5937$

81. $y = \log_2(x+2) = \frac{\log(x+2)}{\log 2}$



3.4	Solving Exponential Equations	https://youtu.be/5cY1ywRQNiA	Pgs. 488 – 491: 2-22 even, 23-47 odd

2.
$$3^x = 81$$
$$3^x = 3^4$$
$$x = 4$$
The solution set is $\{4\}$.

4.
$$5^x = 625$$
$$5^x = 5^4$$
$$x = 4$$
The solution set is $\{4\}$.

6.
$$3^{2x+1} = 27$$
$$3^{2x+1} = 3^3$$
$$2x+1 = 3$$
$$2x = 2$$
$$x = 1$$
The solution set is $\{1\}$.

8.
$$5^{3x-1} = 125$$
$$5^{3x-1} = 5^3$$
$$3x-1 = 3$$
$$3x = 4$$
$$x = \frac{4}{3}$$
The solution set is $\left\{\frac{4}{3}\right\}$

10.
$$4^x = 32$$
$$\left(2^2\right)^x = 2^5$$
$$2^{2x} = 2^5$$
$$2x = 5$$
$$x = \frac{5}{2}$$
The solution set is $\left\{\frac{5}{2}\right\}$.

12.
$$125^x = 625$$
$$\left(5^3\right)^x = 5^4$$
$$5^{3x} = 5^4$$
$$3x = 4$$
$$x = \frac{4}{3}$$
The solution set is $\left\{\frac{4}{3}\right\}$.

14.
$$5^{2-x} = \frac{1}{125}$$
$$5^{2-x} = \frac{1}{5^3}$$
$$5^{2-x} = 5^{-3}$$
$$2-x = -3$$
$$-x = -5$$
$$x = 5$$
The solution set is $\{5\}$.

16.
$$7^{\frac{x-2}{6}} = \sqrt{7}$$
$$7^{\frac{x-2}{6}} = 7^{\frac{1}{2}}$$
$$\frac{x-2}{6} = \frac{1}{2}$$
$$2(x-2) = 6(1)$$
$$2x-4 = 6$$
$$2x = 10$$
$$x = 5$$
The solution set is $\{5\}$.

18.
$$9^x = \frac{1}{\sqrt[3]{3}}$$
$$\left(3^2\right)^x = \frac{1}{3^{\frac{1}{3}}}$$
$$3^{2x} = 3^{-\frac{1}{3}}$$
$$2x = -\frac{1}{3}$$
$$x = \frac{1}{2}\left(-\frac{1}{3}\right) = -\frac{1}{6}$$
The solution set is $\left\{-\frac{1}{6}\right\}$.

20.
$$8^{1-x} = 4^{x+2}$$
$$\left(2^3\right)^{1-x} = \left(2^2\right)^{x+2}$$
$$2^{3-3x} = 2^{2x+4}$$
$$3-3x = 2x+4$$
$$-5x = 1$$
$$x = -\frac{1}{5}$$
The solution set is $\left\{-\frac{1}{5}\right\}$.

22.
$$e^{x+4} = \frac{1}{e^{2x}}$$
$$e^{x+4} = e^{-2x}$$
$$x+4 = -2x$$
$$3x = -4$$
$$x = -\frac{4}{3}$$
The solution set is $\left\{-\frac{4}{3}\right\}$.

23.
$$10^x = 3.91$$
$$\ln 10^x = \ln 3.91$$
$$x \ln 10 = \ln 3.91$$
$$x = \frac{\ln 3.91}{\ln 10} \approx 0.59$$

25.
$$e^x = 5.7$$
$$\ln e^x = 5.7$$
$$x = \ln 5.7 \approx 1.74$$

27.
$$5^x = 17$$
$$\ln 5^x = \ln 17$$
$$x \ln 5 = \ln 17$$
$$x = \frac{\ln 17}{\ln 5} \approx 1.76$$

29.
$$5e^x = 23$$
$$e^x = \frac{23}{5}$$
$$\ln e^x = \ln \frac{23}{5}$$
$$x = \ln \frac{23}{5} \approx 1.53$$

31.
$$3e^{5x} = 1977$$
$$e^{5x} = 659$$
$$\ln e^{5x} = \ln 659$$
$$x = \frac{\ln 659}{5} \approx 1.30$$

33.
$$e^{1-5x} = 793$$
$$\ln e^{1-5x} = \ln 793$$
$$(1-5x)(\ln e) = \ln 793$$
$$1-5x = \ln 793$$
$$5x = 1 - \ln 793$$
$$x = \frac{1-\ln 793}{5} \approx -1.14$$

35.
$$e^{5x-3} - 2 = 10{,}476$$
$$e^{5x-3} = 10{,}478$$
$$\ln e^{5x-3} = \ln 10{,}478$$
$$(5x-3)\ln e = \ln 10{,}478$$
$$5x - 3 = \ln 10{,}478$$
$$5x = \ln 10{,}478 + 3$$
$$x = \frac{\ln 10{,}478 + 3}{5} \approx 2.45$$

37.
$$7^{x+2} = 410$$
$$\ln 7^{x+2} = \ln 410$$
$$(x+2)\ln 7 = \ln 410$$
$$x + 2 = \frac{\ln 410}{\ln 7}$$
$$x = \frac{\ln 410}{\ln 7} - 2 \approx 1.09$$

39.
$$7^{0.3x} = 813$$
$$\ln 7^{0.3x} = \ln 813$$
$$0.3x \ln 7 = \ln 813$$
$$x = \frac{\ln 813}{0.3 \ln 7} \approx 11.48$$

41.
$$5^{2x+3} = 3^{x-1}$$
$$\ln 5^{2x+3} = \ln 3^{x-1}$$
$$(2x+3)\ln 5 = (x-1)\ln 3$$
$$2x \ln 5 + 3\ln 5 = x \ln 3 - \ln 3$$
$$3\ln 5 + \ln 3 = x \ln 3 - 2x \ln 5$$
$$3\ln 5 + \ln 3 = x(\ln 3 - 2\ln 5)$$
$$\frac{3\ln 5 + \ln 3}{\ln 3 - 2\ln 5} = x$$
$$-2.80 \approx x$$

43.
$$e^{2x} - 3e^x + 2 = 0$$
$$(e^x - 2)(e^x - 1) = 0$$
$$e^x - 2 = 0 \quad \text{or} \quad e^x - 1 = 0$$
$$e^x = 2 \qquad e^x = 1$$
$$\ln e^x = \ln 2 \qquad \ln e^x = \ln 1$$
$$x = \ln 2 \qquad x = 0$$

The solution set is {0, ln 2). The solutions are 0 and approximately 0.69.

45.
$$e^{4x} + 5e^{2x} - 24 = 0$$
$$(e^{2x} + 8)(e^{2x} - 3) = 0$$
$$e^{2x} + 8 = 0 \quad \text{or} \quad e^{2x} - 3 = 0$$
$$e^{2x} = -8 \qquad e^{2x} = 3$$
$$\ln e^{2x} = \ln(-8) \qquad \ln e^{2x} = \ln 3$$
$$2x = \ln(-8) \qquad 2x = \ln 3$$

$\ln(-8)$ does not exist $\qquad x = \frac{\ln 3}{2}$

$$x = \frac{\ln 3}{2} \approx 0.55$$

47. $3^{2x} + 3^x - 2 = 0$

$(3^x + 2)(3^x - 1) = 0$

$3^x + 2 = 0$	$3^x - 1 = 0$
$3^x = -2$	$3^x = 1$
$\log 3^x = \log(-2)$	$\log 3^x = \log 1$
does not exist	$\log 3 = 0$
	$x = \dfrac{0}{\log 3}$
	$x = 0$

The solution set is $\{0\}$.

3.4A	Solving Logarithmic Equations	https://youtu.be/iGCTM942ffQ	Pgs. 488 – 491: 54-64 even, 75-89 odd, 92, 96, 100

54. $\log_5(x-7)=2$

$5^2 = x-7$

$32 = x$

56. $\log_7(x+2)=-2$

$7^{-2} = x+2$

$\frac{1}{49} = x+2$

$-1\frac{48}{49} = x$

$-1.98 \approx x$

58. $\log_2(4x+1)=5$

$2^5 = 4x+1$

$32 = 4x+1$

$31 = 4x$

$\frac{31}{4} = x$

$7.75 = x$

60. $6\ln 2x = 30$

$\ln 2x = 5$

$e^{\ln 2x} = e^5$

$2x = e^5$

$x = \frac{e^5}{2} \approx 74.21$

62. $7+3\ln x = 6$

$3\ln x = -1$

$\ln x = -\frac{1}{3}$

$e^{\ln x} = e^{-1/3}$

$x = e^{-1/3} \approx 0.72$

64. $\ln\sqrt{x+4} = 1$

$e^{\ln\sqrt{x+4}} = e^1$

$\sqrt{x+4} = e$

$x+4 = e^2$

$x = e^2 - 4 \approx 3.39.$

75. $\log(x+4) = \log x + \log 4$

$\log(x+4) = \log 4x$

$x+4 = 4x$

$4 = 3x$

$x = \frac{4}{3}$

This value is rejected. The solution set is $\left\{\frac{4}{3}\right\}$.

77. $\log(3x-3) = \log(x+1) + \log 4$

$\log(3x-3) = \log(4x+4)$

$3x-3 = 4x+4$

$-7 = x$

This value is rejected. The solution set is $\{\ \}$.

79. $2\log x = \log 25$

$\log x^2 = \log 25$

$x^2 = 25$

$x = \pm 5$

-5 is rejected. The solution set is $\{5\}$.

81. $\log(x+4) - \log 2 = \log(5x+1)$

$\log\frac{x+4}{2} = \log(5x+1)$

$\frac{x+4}{2} = 5x+1$

$x+4 = 10x+2$

$-9x = -2$

$x = \frac{2}{9}$

$x \approx 0.22$

83. $2\log x - \log 7 = \log 112$

$\log x^2 - \log 7 = \log 112$

$\log\frac{x^2}{7} = \log 112$

$\frac{x^2}{7} = 112$

$x^2 = 784$

$x = \pm 28$

-28 is rejected. The solution set is $\{28\}$.

85. $\log x + \log(x+3) = \log 10$

$\log(x^2+3x) = \log 10$

$x^2+3x = 10$

$x^2+3x-10 = 0$

$(x+5)(x-2) = 0$

$x = -5$ or $x = 2$

-5 is rejected. The solution set is $\{2\}$.

87. $\ln(x-4) + \ln(x+1) = \ln(x-8)$

$\ln(x^2-3x-4) = \ln(x-8)$

$x^2-3x-4 = x-8$

$x^2-4x+4 = 0$

$(x-2)(x-2) = 0$

$x = 2$

2 is rejected. The solution set is $\{\ \}$.

89. $\ln(x-2) - \ln(x+3) = \ln(x-1) - \ln(x+7)$

$\ln\frac{x-2}{x+3} = \ln\frac{x-1}{x+7}$

$\frac{x-2}{x+3} = \frac{x-1}{x+7}$

$(x-2)(x+7) = (x+3)(x-1)$

$x^2+5x-14 = x^2+2x-3$

$3x = 11$

$x = -5$ or $x = 2$

-5 is rejected. The solution set is $\{2\}$.

$x = 2$

2 is rejected. The solution set is $\{\ \}$.

$x^2+5x-14 = x^2+2x-3$

$3x = 11$

$x = \frac{11}{3}$

$x \approx 3.67$

92. $3^{x+2} \cdot 3^x = 81$

$3^{(x+2)+x} = 3^4$

$3^{2x+2} = 3^4$

$2x+2 = 4$

$2x = 2$

$x = 1$

96. $5^{x^2} = 50$

$\ln 5^{x^2} = \ln 50$

$x^2 \ln 5 = \ln 50$

$x^2 = \frac{\ln 50}{\ln 5}$

$x = \pm\sqrt{\frac{\ln 50}{\ln 5}} \approx \pm 1.56$

100. $3^{x^2-12} = 9^{2x}$

$3^{x^2-12} = \left(3^2\right)^{2x}$

$3^{x^2-12} = 3^{4x}$

$x^2-12 = 4x$

$x^2-4x-12 = 0$

$(x-6)(x+2) = 0$

Apply the zero product property:

$x-6 = 0$ or $x+2 = 0$

$x = 6$ $\quad$ $x = -2$

The solution set is $\{-2, 6\}$.

3.5	Exponential Growth and Decay Functions Day 1	https://youtu.be/rrAIGbUemsg	3.5 Worksheet

PC Exponential Growth and Decay WS #1 Name_____ out of 22

The exponential models describe the population of the indicated country, A, in millions, t years after 2006. Use these models to solve Exercises 1-5.

India: $A = 1095.4e^{0.014t}$

Iraq: $A = 26.8e^{0.027t}$

Japan: $A = 127.5e^{0.001t}$

Russia: $A = 142.9e^{-0.004t}$

1. What was the population of Japan in 2006? 127.5 million
2. What was the population of Iraq in 2006? 26.8 million
3. Which country has the greatest growth rate? By what percentage is the population of that country increasing each year? Iraq, 2.7%
4. Which country has a decreasing population? By what percentage is the population of that country decreasing each year? Russia, -.4%
5. When will India's population be 1238 million? (2 points)

$$t = \frac{\ln\left(\frac{1238}{1095.4}\right)}{.014} \approx 8.7 \text{ years}$$

2014

6. The half-life of radium is 1690 years. If 10 grams are present now, how much will be present in 50 years? (2 points)

$A = 10e^{k(50)}$ $\quad k = \frac{\ln(.5)}{1690} \approx -.00041$ $\quad A = 10e^{k(50)} \approx 9.8$ grams

7. The half-life of radioactive potassium is 1.3 billion years. If 10 grams are present now, how much will be present in 1,000 years? In 1,000,000 years? (2 points)

$k = \frac{\ln(.5)}{1{,}300{,}000{,}000} \approx -.0000533$

$A = 10e^{k(1000)} = 9.999995$ grams

$A = 10e^{k(1{,}000{,}000)} = 9.995$ grams

8. A piece of charcoal is found to contain 30% of the carbon-14 it originally had. When did the tree from which the charcoal came die? Use 5,715 years as the half-life of carbon-14. (2 points)

$t = \frac{\ln(.30)}{k} \approx$ 9,926.8 years ago

$k = \frac{\ln(.5)}{5715} \approx -.000121$

9. A radioactive substance has a half-life of 420 years. How much remains of a 2 ounce sample after 200 years?

$k = \frac{\ln(.5)}{420} \approx -.00165$

$A = 2e^{k(200)} \approx$ 1.44 oz

10. An isotope of sodium has a half-life of 15 hours. How many hours will it take for 40% of a given amount to remain?

$k = \frac{\ln(.5)}{15} = -.0462$

$t = \frac{\ln(.40)}{k}$

$\approx$ 19.8 hours

Use the following formula ***t= ln2/k*** *that gives the time for a population with a growth rate k to double to solve the following. Express each answer to the nearest whole year.*

11. The growth model $A=4.1e^{0.01t}$ describes New Zealand's population, A, in millions, *t* years after 2006.

a. What is New Zealand's growth rate? .01 or 1%

b. How long will it take New Zealand to double its population?

$t = \frac{\ln(2)}{.01} \approx$ 69 years

c. How long will it take New Zealand to triple its population?

$t = \frac{\ln(3)}{.01} \approx$ 110 years

12. You open a savings account that pays 6.5% annual interest. After how many years will you have twice the amount you deposited? Assume continuous compounding.

$FV = Pe^{rt}$

$t = \frac{\ln(2)}{.065} \approx$ 10.7 years

13. How long will it take money to triple in an account that pays 8.1% in interest and is compounded continuously?

$t = \frac{\ln(3)}{.081} \approx$ 13.6 years

14. If you deposit $1,000 into an account that pays 9% annual interest, compounded monthly, how long would you have to wait to have $5,000 in your account? (2 points)

$$5000 = 1000\left(1+\frac{.09}{12}\right)^{12t}$$

$$\ln(5) = \ln\left(1+\frac{.09}{12}\right)^{12t}$$

$$t = \frac{\ln(5)}{12\ln(1+.09/12)} \approx 17.9 \text{ years}$$

15. How much money must be invested at 9% interest compounded continuously to yield $650 after 2 years?

$$P = \frac{650}{e^{.09(2)}} \approx \$542.93$$

3.5A	Exponential Growth and Decay Functions Day 2	https://youtu.be/1SOvLZtNM7M	3.5A Worksheet

PC Exponential Growth and Decay WS #2 Name_____ out of 30
(3 points each)

1. A bacteria culture has an initial count estimate of 4,000. After 20 minutes the count is 22,400. Approximately how many minutes did it take for the culture to double?

8.047 min

2. The population of a colony of mosquitoes obeys the law of uninhibited growth. If there are 1,000 mosquitoes initially, and there are 1,800 after 1 day, a) what is the size of the colony after 3 days? b) How long is it until there are 10,000 mosquitoes?

$A = A_0 e^{kt}$
$= 1000 e^{k(3)}$
$k = \frac{\ln(18/10)}{1}$
$\approx .587786$

$t = \frac{\ln(10)}{.5877}$

a) 5,832 b) 3.917 days

3. A culture of bacteria obeys the law of uninhibited growth. If there are 500 bacteria cells present initially, and there are 800 after 1 hour, how many will be present in the culture after 5 hours?

$A = A_0 e^{kt}$
$= 500 e^{k(5)}$

$k = \frac{\ln(8/5)}{1}$

5,242.88

4. Using the information in #3, how long until there are 20,000 bacteria?

$t = \frac{\ln(40)}{.470004}$

7.849 hrs

5. The population of a southern city follows the exponential law. If the population doubled in size over an 18 month period and the current population is 10,000, what will the population be 2 years from now?

$A = A_0 e^{kt}$
$10{,}000 e^{k(24)}$

$k = \frac{\ln(2)}{18} \approx$

25,198.421

6. The population of a Midwestern city follows the exponential law. If the population decreased from 900,000 to 800,000 from 1984 to 1986, what will the population be in 1988 assuming the exponential model is followed?

$k = \frac{\ln(8/9)}{2} \approx -.0588\ldots$

$A = A_0 e^{kt}$
$= 900000 e^{k(4)}$

711,111.1

8. Suppose a certain radioactive substance has a half-life of 276 hours. How long will it take for only 30% of the substance to remain?

$t = \frac{\ln(.3)}{k}$

$k = \frac{\ln(.5)}{276}$

479.4 hrs

9. Kyle deposits $900 into an account earning 5.6% annual interest with monthly compounding. In how many years will he have triple his original deposit?

$n = 12$

$A = A_0\left(1+\frac{r}{n}\right)^{nt}$

$t = \frac{\ln(\text{Prop})}{n\ln\left(1+\frac{r}{n}\right)}$

$= \frac{\ln(3)}{12\ln\left(1+\frac{.056}{12}\right)}$

19.66 yrs

10. Julia has tripled her savings after 10 years. What is her fixed annual interest rate assuming continuous compounding?

$\approx 10.99\%$

11) 20.514 hours

$t = \frac{\ln(2)}{k}$

$k = \frac{\ln(3/2)}{12} =$

3.5B	Newton's Law of Cooling	https://youtu.be/eC2Bdr6M_1o	3.5B Worksheet

1) How much money must be invested at 8.7% interest compounded continuously to yield $4,555 after 5 years? (2 points)

$A_0 = \frac{A}{e^{kt}}$

$2,948.29

2) How long must an amount of money be invested at 9% interest compounded monthly before it is doubled? (3 points)

$A = A_0(1 + r/n)^{nt}$

$t = \frac{\ln\left(\frac{A}{A_0}\right)}{n(1+r/n)}$ $t = \frac{\ln(2)}{\left(12\left(1+\frac{.09}{12}\right)\right)}$

7.73 yrs

3) An exponential model for the population in Russia is given by the following. A in millions, t years after 2006. $A = 142.9e^{-.004t}$ (4 points)

a) What was the population in 2006? — a) 142.9 million

b) Is the population decreasing or increasing? — b) dec

c) What will the population be in 2014? — c) 138.4 million

d) When will the population be 129 million according to the model? — d) 25.58 yrs

$t = \frac{\ln(129/142.9)}{-.004}$

2031

Carbon Dating 4) A fossil is found to contain 32% of its original amount of carbon-14. How old is the fossil? Use 5,715 years as the half-life. (2 points)

$\frac{\ln(.32)}{-}$ $k = \frac{\ln(.5)}{5715}$

9,394.64 yrs

5) A certain radioactive substance decays to 28% of its original amount in 145 years. Find its half-life. (2 points)

$t = \frac{\ln(.5)}{k}$ $k = \frac{\ln(.28)}{145}$

78.95 yrs

6) Liberty and Kailey were strolling through the park one day at 4:12 pm on a perfect 79 degree spring day. Suddenly, they discovered a dead body. Liberty took out her handy dandy thermometer and measured the temperature of the body as 97.7 degrees. One hour later, she measured the temperature and it was 96.2 degrees. At what time did the body die? Assume normal body temperature is 98.6 degrees. Use Newton's Law of Cooling. (3 points)

$$t = \frac{\ln\left(\frac{97.7-79}{98.6-79}\right)}{k} \qquad k = \frac{\ln\left(\frac{96.2-79}{97.7-79}\right)}{60} \qquad k \approx -.001393$$

$t = 93.74$ min

5:12

3:38pm

7) A thermometer reading 72 degrees F is placed in a refrigerator where the temperature is a constant 38 degrees F.

a) If the thermometer reads 60 degrees F after 2 minutes, what will it read after 7 minutes? (2 points)

$$t_f = (t_0 - t_r)e^{kt} + t_r$$
$$= (72-38)e^{k(7)} + 38$$

$$k = \frac{\ln\left(\frac{60-38}{72-38}\right)}{2}$$

45.41°F

b) How long will it take before the thermometer reads 39 degrees F? (1 point)

$t = \ln($

16.20min 16min 12sec

8) A pizza baked at 450 degrees F is removed from the oven at 5 pm into a room that is a constant 70 degrees F. After 5 minutes, the pizza is 300 degrees F. At what time can you begin eating the pizza if you want its temperature to be 135 degrees F? (2 points)

$$t = \frac{\ln\left(\frac{135-70}{450-70}\right)}{k} \qquad k = \frac{\ln\left(\frac{300-70}{450-70}\right)}{5}$$

$t \approx 17.6$ $\qquad k \approx -.095085$

5:17pm