Pre-Calculus with TRIG – Unit 3 – Into to Trig

Day 1 – Section 4.1 – Intro to Radians

Objectives: Convert angles from radians to degrees and vice-versa. Find the arc length of a circle. Describe the meaning of a radian measure.

Review Questions of the day:

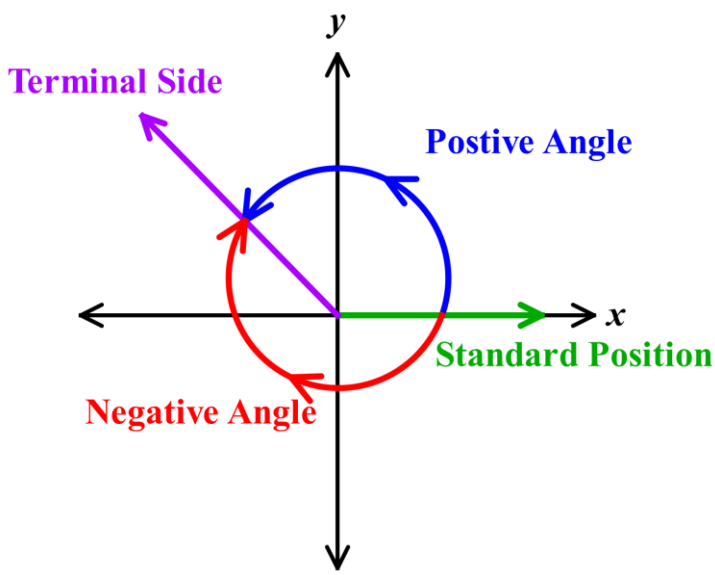
1) Solve for all values of x . $x^2 - 7x + 6 = 0$

2) What is the circumference of a circle with a radius of 6 in.?

Trigonometry is the study of _____ and _____.

An angle can be measured in either _____ or _____.

Standard Position of an Angle:

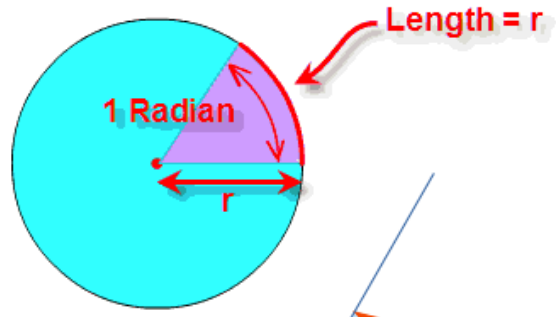


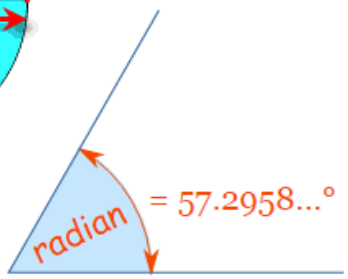
Terminal Side:

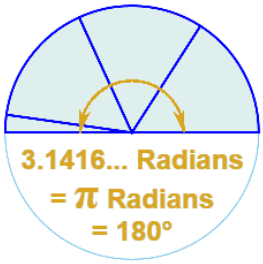
Positive Angle versus Negative Angle

Radians:

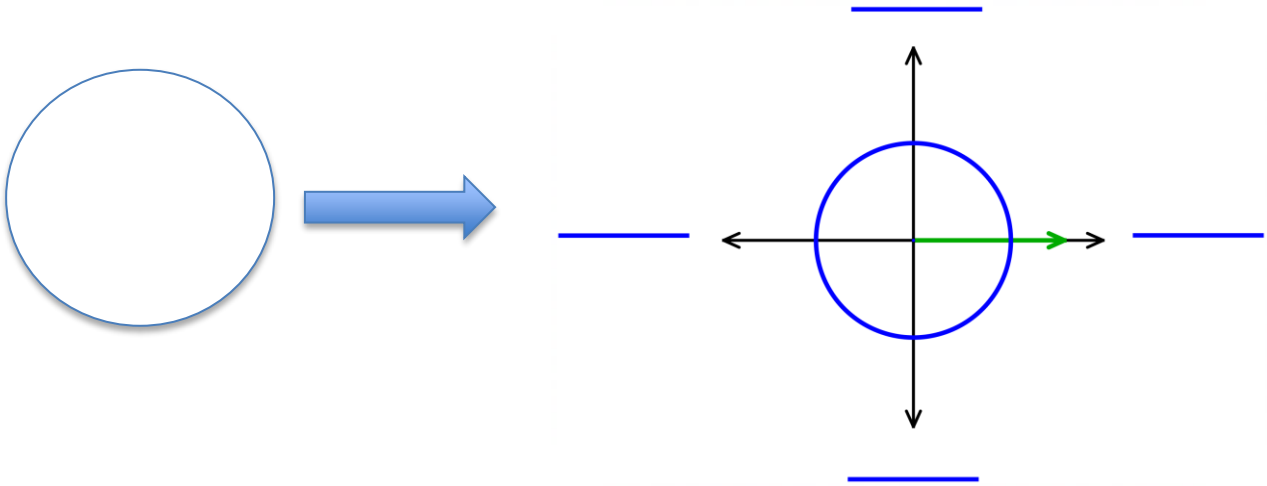
- One radian = the angle of an arc created by wrapping the radius of a circle around its circumference.
- There are 2π radians in a unit circle.
- One Radian = **$180/\pi$ degrees**, or about 57.296°
- So, a **Radian** "cuts out" a length of a circle's Circumference equal to the radius.







The best way to understand the measure of a radian is to think of the meaning of π. What is the meaning of 2π?



How many diameters can wrap around a circle? _____ So, how many radii can wrap around a circle? _____

That is why there are always _____ radians in a circle but we use this term as a unit that measures an _____, not the arc length or circumference. The arc length is measured in terms of a measure of length such as inches or feet but the angle is measured in terms of radians.

Radians ↔ Degrees Conversion

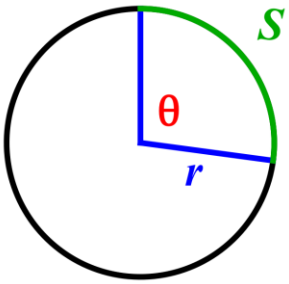
Convert each to radians.

1) $180°$ **2)** $-27°$ **a)** $105°$ **b)** $57.3°$

Convert each to degrees.

5) $\frac{\pi}{4}$ **6)** $\frac{-4\pi}{3}$ **7)** 2.7 **c)** $\frac{5\pi}{3}$ **d)** 1.57

Arc Length:



Find the length of the arc on a circle of radius r intercepted by a central angle of θ

8) $r = 2.2cm$ $\theta = \frac{2\pi}{3}$

9) $r = 14\ inches$ $\theta = 4.2$

10) $r = 12cm$ $Central\ Angle = 60°$

e) $r = 7ft$ $\theta = 117°$

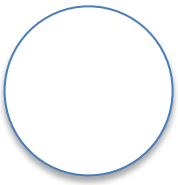
Find the radian measure of the central angle of a circle of radius r that intercepts an arc length of S.

11) r = 12 inches S = 24 inches

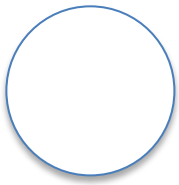
f) r = 3 meters S = 1800 cm

Find the degree measure of a central angle of a circle of radius r that intercepts an arc length of s.

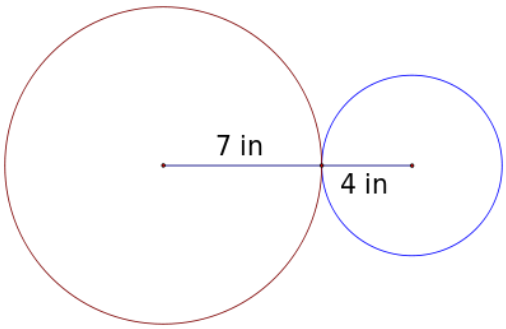
12) r = 11 meters and s = 286 meters



g) r = 12 meters and s = 1.5 meters



13) Two connected gears are rotating. The smaller gear has a radius of 4 inches and the larger gear's radius is 7 inches. What is the angle (in radians) through which the larger gear has rotated when the smaller gear has made one complete rotation?



Day 2 – Section 4.1A – Coterminal Angles

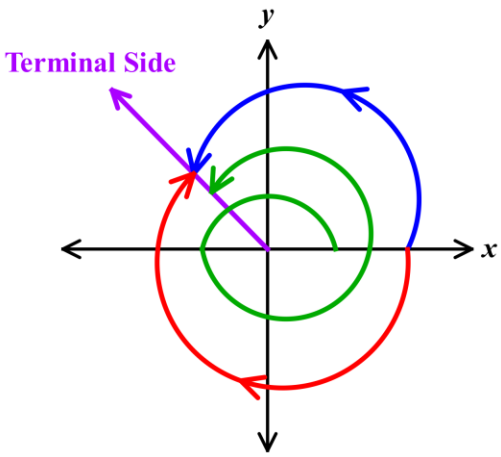
Objectives: Find Coterminal angles, and fill out Unit Circle's degrees and radian measure.

Review Questions of the day:

1) Change $\frac{5\pi}{6}$ to degrees

2) Find a coterminal angle for 360 degrees.

3) How many degrees are in exactly one radian?



Coterminal Angles:

Find a positive and negative angle less than one revolution that is coterminal with the given angle.

1) $400°$ **2)** $-900°$ **a)** $-175°$

Positive: _____ **Positive:** _____ **Positive:** _____

Negative: _____ **Negative:** _____ **Negative:** _____

In order to find a coterminal angle when the angle is in degrees, _____.

Find a positive angle less than one revolution that is coterminal with the given angle.

4) $\frac{12\pi}{5}$ **5)** $\frac{23\pi}{6}$ **6)** $-\frac{9\pi}{4}$ **7)** 9.89 **b)** $\frac{51\pi}{6}$

In order to find a coterminal angle when the angle is in radians, _____.

8) Find a positive coterminal angle between 0° and 360° for 114,567°

c) 1,677°

Find the positive radian measure of the angle that the second hand of a clock moves through in the given time.

10) 50 seconds

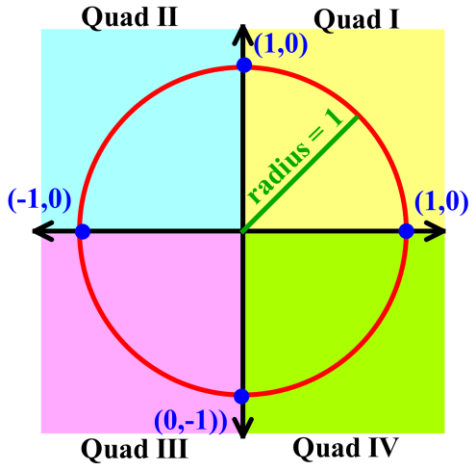
11) 5 minutes and 30 seconds

d) 6 minutes and 24 seconds

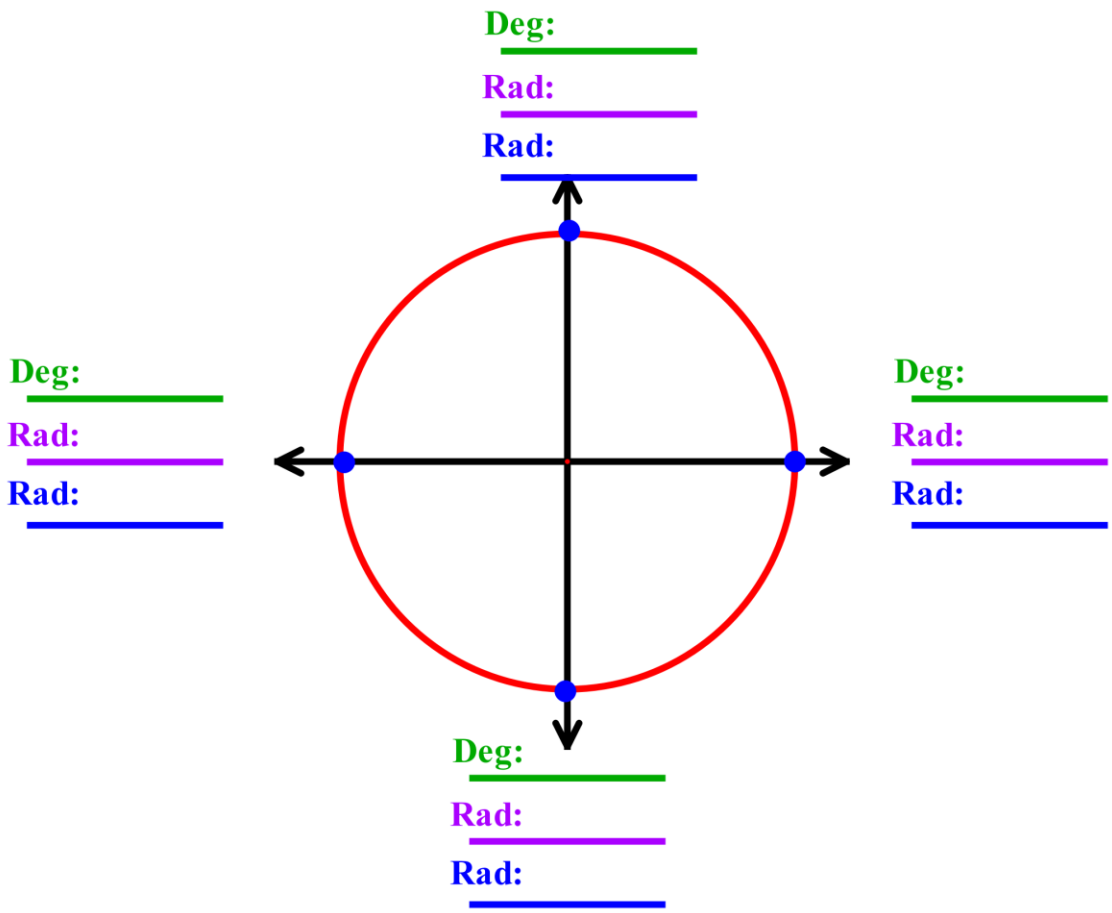
Important Formulas to know…

Coterminal Angles: Land in same position on circle, same terminal side…$+360°k$ or $+2\pi k$ where k is an integer

Unit Circle



Label the following unit circle in degrees, radians in exact form (π), and radians in decimal form.



In which quadrant does the terminal side of each angle lie?

13) 2.3 **14)** $-390°$ **15)** $\frac{5\pi}{3}$ **e)** -6.00 **f)** $-1134°$ **g)** 1.67π

Now turn to the very back page of your guided notes packet, and add the degrees, and radian measure to the Blank Unit Circle.

Day 3 – Section 4.1B – Angular and Linear Velocity

Objectives: Students will be able to calculate **linear and angular velocity**..

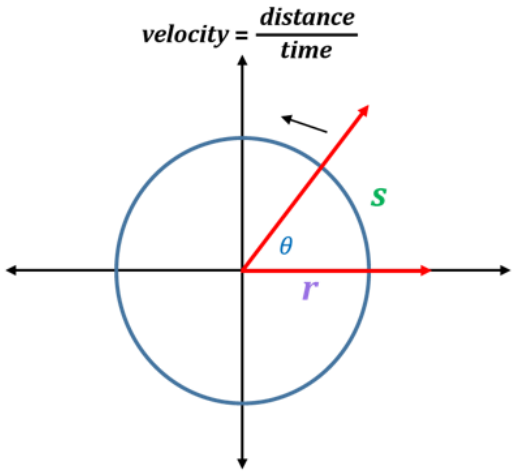
Review Questions of the day:

1) Find a negative coterminal angle for 230°.

2) Change 120° to radians. Leave in π form.

3) Solve $2e^{3x} = 32$ Leave in exact natural log form.

Consider an object or person that moves in a circular manner, such as a person on a carousel, a person on a Ferris wheel, or a point on a CD.



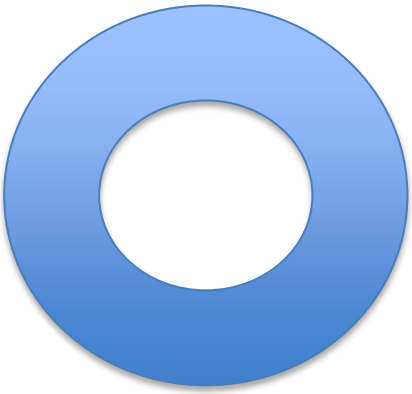
Linear Velocity

Two speeds occur when circular motion is considered:

Types of Circular Motion		
Type	**Definition**	**Formulas**
Angular Speed	this is the **number of radians** the object or person travels per unit of time $\left(\frac{radians}{time}\right)$	
Linear Speed	some **distance** traveled per unit of time $\left(\frac{feet,\ miles,\ etc}{time}\right)$	

1) Emma and Cora are riding on a carousel in San Francisco. Emma is somewhat cautious and wants to ride near the center, 8 feet from the center. Cora would rather sit near the edge 18 feet from the center. The carousel is rotating 2.5 revolutions per minute.

a) Find the angular speed for Emma and for Cora.



b) Find the linear speed for Emma and for Cora.

2) Carson is riding on a Ferris wheel with a radius of 30 feet. The wheel is rotating at 1.5 revolutions per minute. Find the angular and linear speed in feet per minute of Carson's seat on the Ferris wheel.

3) NO IPODS in the 80's: Consider old records. There were the large records, called 33 1/3's and the small records, called 45's. Find the angular velocity for each.

Degrees, Minutes, and Seconds (DMS)

Surveyors measure angles in degrees, minutes, and seconds because they need very accurate measures.

60 minutes = 1 degree
60 seconds = 1 minute
3600 seconds = 1 degree

Convert each to DMS. Round to the nearest second.

4) $23.46°$

a) $47.58°$

Convert each to decimal form. Round to two decimal places.

6) $34°17'23''$

b) $68°30'25''$

FORMULAS TO KNOW:

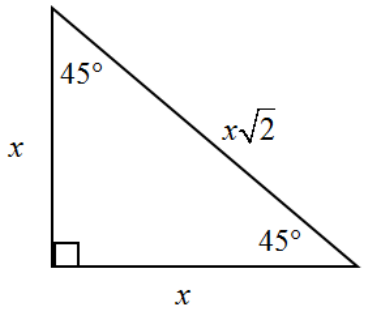
In order to find **angular velocity,** simply multiply by _____. The symbol used for this is _____. This always represents _____ per unit of time.

In order to find **linear velocity (v)**, use the formula v = _____. This always represents _____ per unit of time.

Day 4 – Section 4.2 – Constructing the Unit Circle Part I – At Quadrantals and $\pi/4$

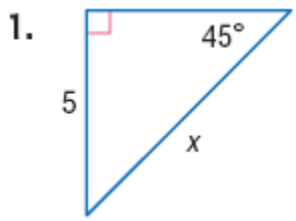
Objectives: Introduce the unit circle and trig values for given angles in any circle. Know what is meant by periodic functions.

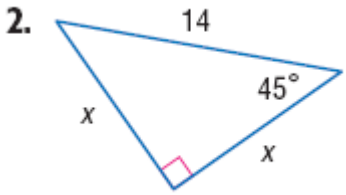
Review Special Right Triangles 45° – 45° – 90°

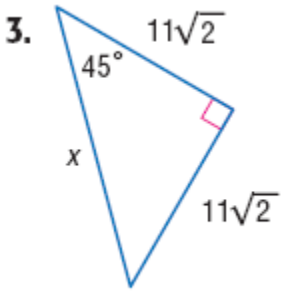


Review Questions of the day:

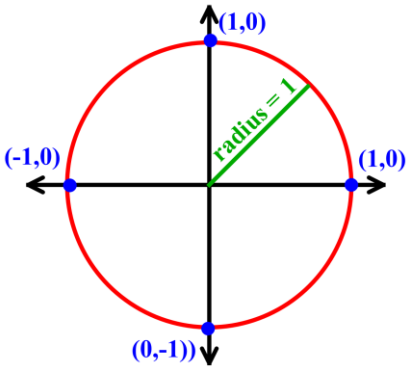
Solve for x and/or y in the following examples.







Unit Circle



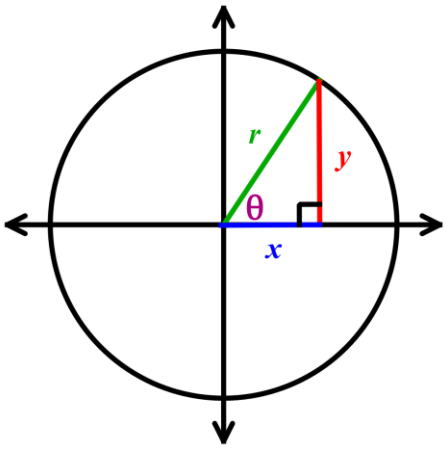
In order to simplify these trig ratios, we use what is called a unit circle. A unit circle has a radius of 1. First, we will look at the **quadrantal angles** and **the $\frac{\pi}{4}$ angles (also called 45's).** Today, we are going to add on a total of eight points on the unit circle to illustrate this idea.

In order to find trig values for quadrantal angles, follow these simple rules/formulas: Let $\theta = angle$

$$\sin(\theta) = \frac{y}{r}$$

$$\cos(\theta) = \frac{x}{r}$$

$$\tan(\theta) = \frac{y}{x}$$

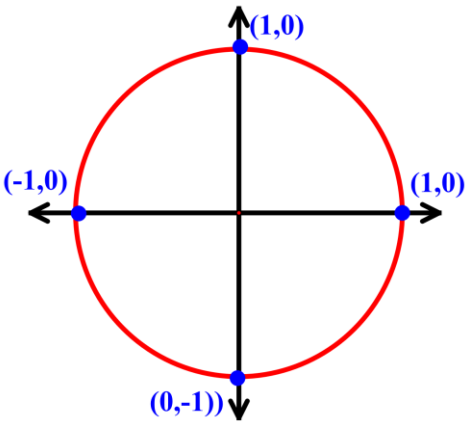


<u>Review of Trig Identities:</u>

Trig Function	Ratio of Sides	Reciprocal Identifies
Sine	$sin\theta =$ _____	
Cosine	$cos\theta =$ _____	
Tangent	$tan\theta =$ _____	
Cosecant	$csc\theta =$ _____	
Secant	$sec\theta =$ _____	
Cotangent	$cot\theta =$ _____	

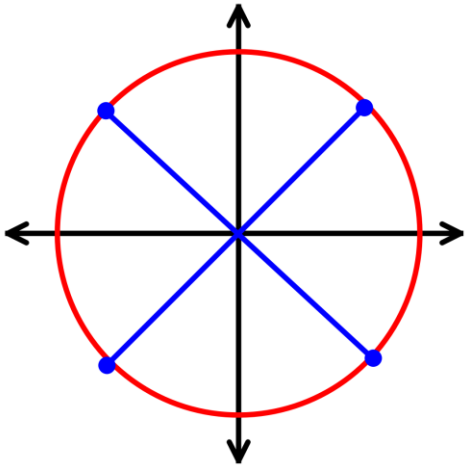
QUADRANTAL TRIG VALUES

	Cosine	Sine	Tangent	Secant	Cosecant	Cotangent
θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
0						
$\frac{\pi}{2}$						
π						
$\frac{3\pi}{2}$						
2π						



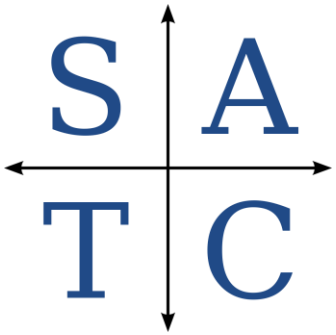
$\pi/4$ TRIG VALUES

	Cosine	Sine	Tangent	Secant	Cosecant	Cotangent
θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\frac{\pi}{4}$						
$\frac{3\pi}{4}$						
$\frac{5\pi}{4}$						
$\frac{7\pi}{4}$						



Now turn to the very back page of your guided notes packet, and add the degrees, and radian measure to the Blank Unit Circle.

How to determine if the value is positive or negative:



$\cos(\theta)$ **$\sin(\theta)$** **$\tan(\theta)$**

For these examples, get a coterminal angle within one revolution and find the exact value of each. Remember, period for cosine and sine is _____ and for tangent, it's _____.

1) $cos\left(-\frac{11\pi}{4}\right)$ **2)** $csc\left(\frac{9\pi}{4}\right)$ **3)** $\sin\left(\frac{13\pi}{4}\right)$ **a)** $tan(3\pi)$

4) $\cos\left(\frac{\pi}{4}+4\pi\right)$ **5)** $sec\left(\frac{3\pi}{4}+18\pi\right)$ **6)** $-tan\left(\frac{7\pi}{4}-11\pi\right)$ **b)** $cot\left(-\frac{\pi}{4}+31\pi\right)$

7) $tan(0+300\pi)$ **8)** $\cos\left(\frac{-23\pi}{4}\right)$ **9)** $sin\left(-\frac{3\pi}{4}+6\pi\right)$ **c)** $csc\left(-\frac{\pi}{4}+14\pi\right)$

10) $sin\left(\frac{\pi}{4}+13\pi\right)+cos\left(\frac{3\pi}{4}+\pi\right)+tan\left(\frac{5\pi}{4}+11\pi\right)$

Any corresponding coterminal for each angle above will have the same trig value.

For $cos(t)$, $sin(t)$, $sec(t)$, and csc(t), the period is 2π This means the trig values will repeat themselves every 2π or every rotation. We say that trig functions are periodic.

cos(t +) = cos(t) sin(t +) = sin(t)

sec(t +) = sec(t) csc(t +) = csc(t)

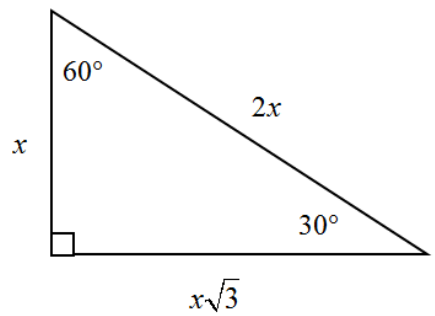
For $tan(t)$ and $cot(t)$, a similar rule applies but the period for each of these is _____.

Tan(t +) = tan(t) Cot(t +) = cot(t)

Day 4 – Section 4.2A – Constructing the Unit Circle Part II – $\pi/6$ and $\pi/3$

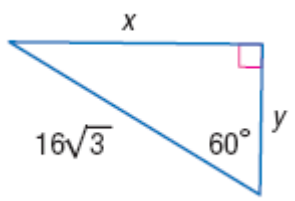
Objectives: Introduce the trig values for $\frac{\pi}{6}$ and $\frac{\pi}{3}$ angles in any circle and determine all trig values for any angle coterminal to these.

Review Special Right Triangles 30° – 60° – 90°

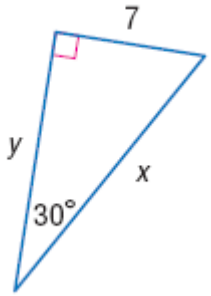


Review Questions of the day:

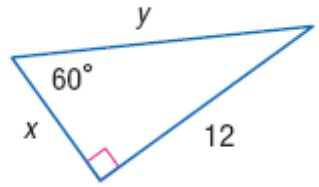
4.



5.

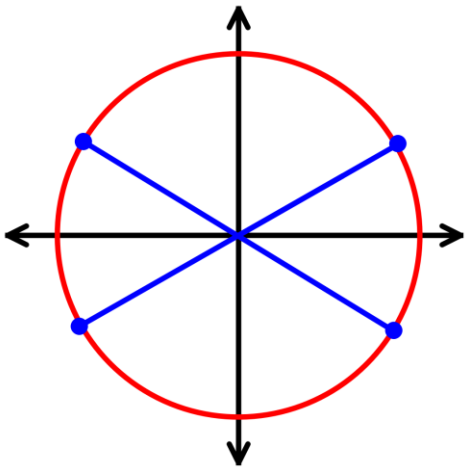


6.



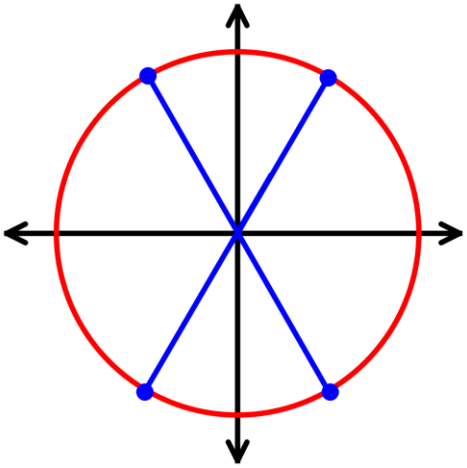
$\pi/6$ TRIG VALUES

	Cosine	Sine	Tangent	Secant	Cosecant	Cotangent
θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\frac{\pi}{6}$						
$\frac{5\pi}{6}$						
$\frac{7\pi}{6}$						
$\frac{11\pi}{6}$						



$\pi/3$ TRIG VALUES

	Cosine	Sine	Tangent	Secant	Cosecant	Cotangent
θ	$\cos(\theta)$	$\sin(\theta)$	$\tan(\theta)$	$\sec(\theta)$	$\csc(\theta)$	$\cot(\theta)$
$\frac{\pi}{3}$						
$\frac{2\pi}{3}$						
$\frac{4\pi}{3}$						
$\frac{5\pi}{3}$						



Combine all of your values at home to make one table on the colored sheet.

Any corresponding coterminal for each angle above will have the same trig value.

Find the following.

1) $\sin\left(\frac{\pi}{3} + 4\pi\right)$

2) $\cos\left(\frac{\pi}{6} + 6\pi\right)$

a) $\cos\left(-\frac{\pi}{3} + 12\pi\right)$

3) $\sin\left(\frac{\pi}{6} - 2\pi\right)$

4) $-\sec\left(-\frac{\pi}{3}\right)$

b) $-\tan\left(-\frac{\pi}{3} + 14\pi\right)$

Even and Odd Functions:

Which two trig functions are even? _____

Compare the following two trig functions for each example.

5) $cos\left(-\frac{\pi}{3}\right) \quad cos\left(\frac{\pi}{3}\right)$

6) $cos\left(-\frac{5\pi}{4}\right) \quad cos\left(\frac{5\pi}{4}\right)$

c) $sec\left(-\frac{\pi}{4}\right) \quad sec\left(\frac{\pi}{4}\right)$

Which two trig functions are odd? _____

Compare the following two trig functions for each example.

7) $sin\left(-\frac{\pi}{6}\right) \quad sin\left(\frac{\pi}{6}\right)$

8) $csc\left(-\frac{5\pi}{4}\right) \quad csc\left(\frac{5\pi}{4}\right)$

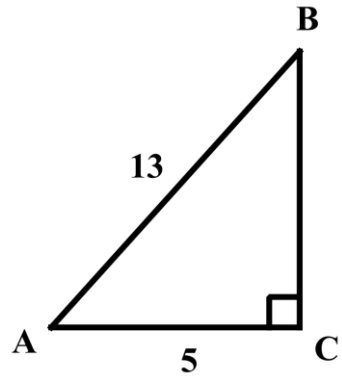
9) $cot(-75°) \quad tan(-75°)$

d) $tan\left(-\frac{5\pi}{6}\right) \quad tan\left(\frac{5\pi}{6}\right)$

Day 5 – Section 4.2B – Trig Identities

Objectives: Introduce and apply basic trig identities.

Review Questions of the day:



1) Find a negative coterminal angle for $-32{,}270°$

2) Find a coterminal angle between $0°$ and $360°$ for $143{,}431°$.

3) Find the sine, cosine, and tangent for $\angle A$.

4) Evaluate, using the values from #3, $\frac{\sin(A)}{\cos(A)}$

Ratio Identities:

$\boldsymbol{tan(A)} =$ _____

$\boldsymbol{cot(A)} =$ _____

Use the ratios above in order to find $\boldsymbol{tan(A)}$ and $\boldsymbol{cot(A)}$.

1) $cos(A) = \frac{4}{5}$ $\quad sin(A) = \frac{3}{5}$

2) $cos(A) = \frac{2}{3}$ $\quad sin(A) = \frac{\sqrt{5}}{3}$

Reciprocal Identities:

$sin(A) =$ _____ $\quad cos(A) =$ _____ $\quad tan(A) =$ _____

$csc(A) =$ _____ $\quad sec(A) =$ _____ $\quad cot(A) =$ _____

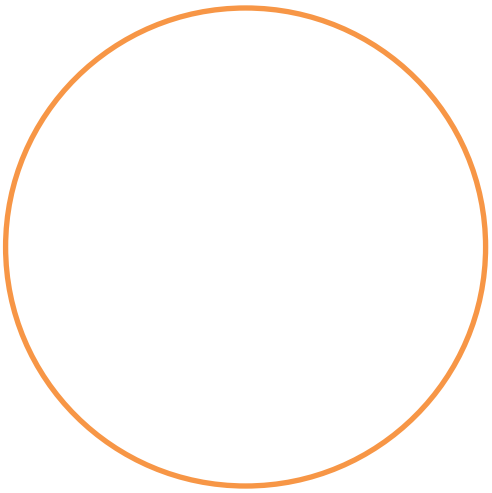
Use the ratios above in order to find the value of each trig expression.

3) $cos(3) \cdot sec(3)$

4) $cot\ (230°)\ tan\ (230°)$

a) $17\left(sin\ \left(\frac{\pi}{7}\right)\right)\left(csc\ \left(\frac{\pi}{7}\right)\right)$

Pythagorean Identities:



From this unit circle, we know that $x^2 + y^2 = 1$

Substitute _____ for *x* and _____ for *y* and this yields…

Main Pythagorean Identity

Divide this equation by _____ and you get…

Tangent Corollary Pythagorean Identity

Now divide the **main** equation by _____ and you get…

Cotangent Corollary Pythagorean Identity

Use the Main Pythagorean Identity above in order to find $sin(A)$. Assume angle A lies in Quad I.

5) $\cos(A) = \frac{5}{6}$

b) $\cos(A) = \frac{\sqrt{17}}{7}$

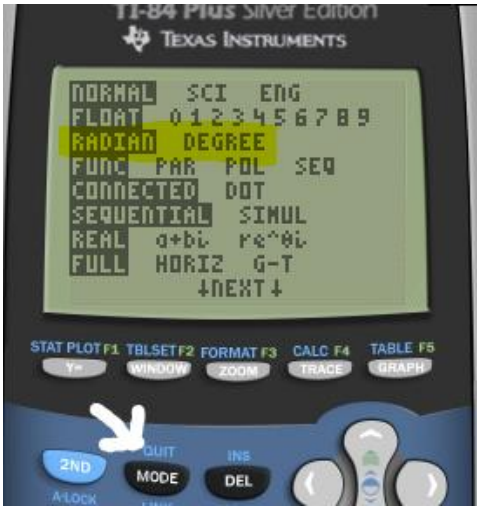
Calculator Work – Must check mode (° → *Degrees*, everything else is radians) when entering a trig function! Round each to four decimal places.

6) $\cos(2.3)$

7) $sin(3.89)$

8) $sec(432°)$

c) $cot\left(\frac{\pi}{7}\right)$



Day 6 – Section 4.3 – Right Triangle Trig

Objectives: Solve for sides and angles in a right triangle, as well as labeling a bearing

Review Questions of the day:

1) Find $sin(4\pi)$.

2) Find the angular velocity for an object spinning 3 revolutions per minute.

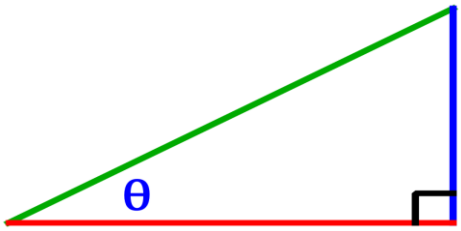
3) What is $cos(0)$?

S O H – C A H – T O A

TRIG FUNCTIONS ARE DERIVED FROM RIGHT TRIANGLES

sin Θ = **cos Θ =** **tan Θ =**

csc Θ = **sec Θ =** **cot Θ =**

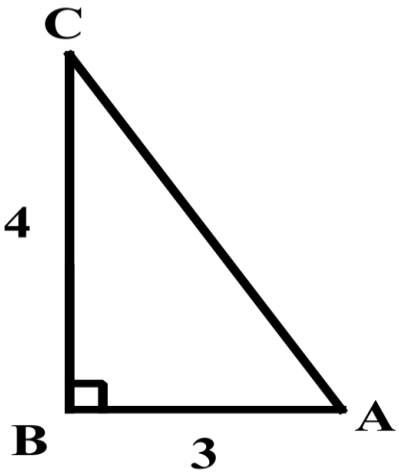


1) Write the 6 trig ratios given the right triangle

sinA cscA

cosA secA

tanA cotA

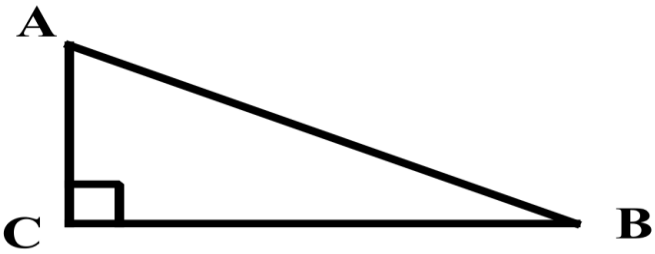


b) Write the 6 trig ratios when given the right triangle
AC = 3 and AB = 7

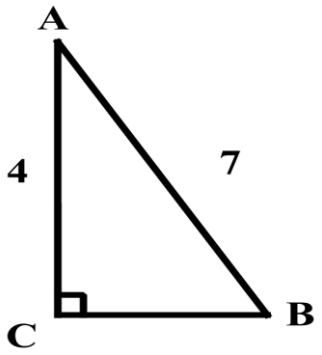
sinB cscB

cosB secB

tanB cosB

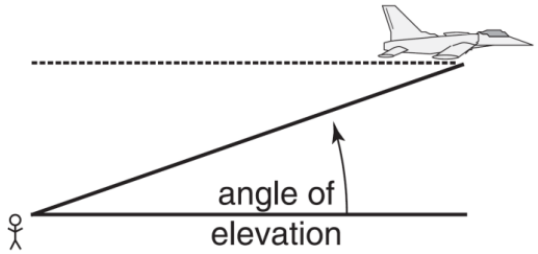


2) Find the $m\angle B$ in decimal form and in DMS.

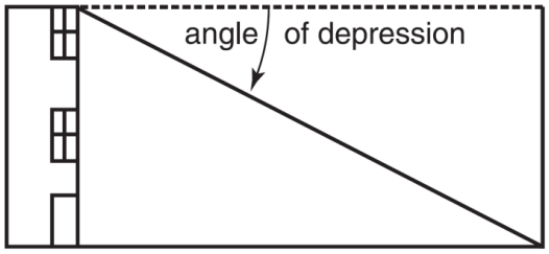


When given any two parts of a right triangle, use _____ to solve the triangle. When you are solving for an angle, use an _____.

Angle of Elevation



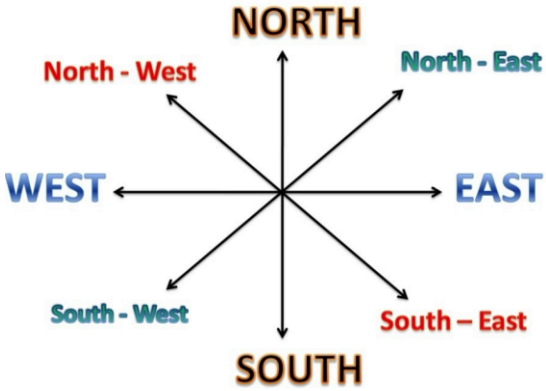
Angle of Depression



3) Ben is looking up at an object on top of a building through an **angle of elevation** of 50^0. If he is 70 feet from the base of the building, how high is the building to the nearest tenth?

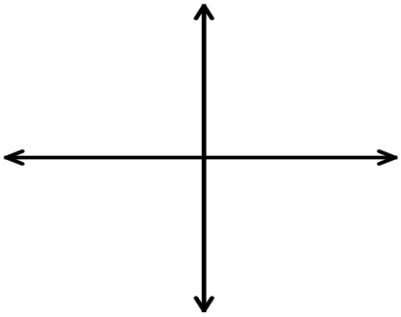
4) Alita walks toward a landmark. Initially, the angle of elevation to the top of the landmark is 52 degrees. She walks a distance of 100 feet. At that point, the **angle of elevation** is 68 degrees. How tall is the landmark to the nearest tenth?

Bearings:

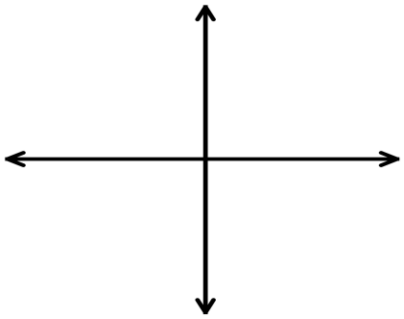


Draw in the following bearings.

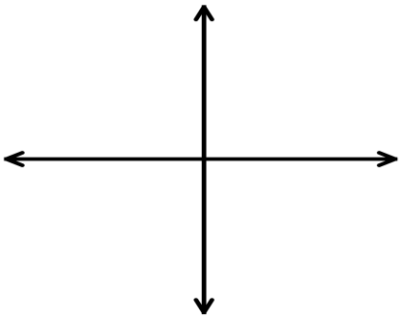
5) N 45^0 W



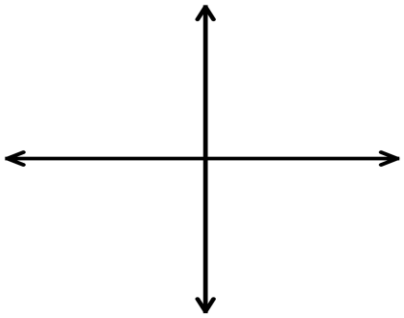
6) N 55^0 E



7) S 35^0 W



b) S 22^0 E



8) A boat leaves the entrance to a harbor and travels 35 miles on a bearing N43^0E. Captain Newbs then turns the boat 90^0 clockwise and travels 19 miles on a bearing S46^0E. At that time:

a) How far is Captain Newbs boat, to the nearest tenth of a mile, from the harbor entrance?

b) What is the bearing, to the nearest second, of the boat from the harbor entrance?

Day 7 – Section 4.3A – Cofunctions and Identities

Objectives: Find Cofunctions and use Basic Trig Identities

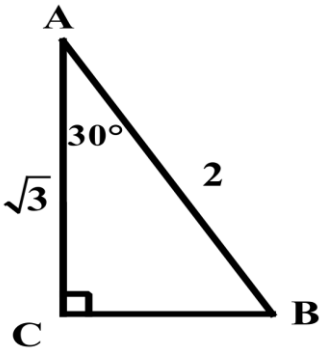
Review Questions of the day:

1) Find a coterminal angle between 0 and 2π for $112\pi/3$.

2) How many radians does the second hand on a clock travel in 6 minutes? Leave in terms of π.

3) Fill in the blank. When finding an angle in a triangle, always use an _____ function.

Exploring relationships in a specific right triangle:

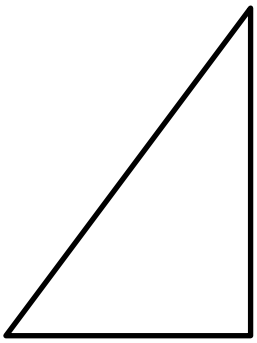


$\sin(30°)$ $\cos(60°)$

$\tan(60°)$ $\cot(30°)$

$\csc(60°)$ $\sec(30°)$

Generalizing this, we know that when we look at the complement, the _____ side becomes the _____ side and vice – versa so…



cos(A) = sin(- A)

tan(A) = cot(- A)

sec(A) = csc(- A)

These are called _____.

Give a cofunction for each. If given degrees, leave answer in degrees and if given radians, leave answer in radians.

1) $\cos(10°)$ **2)** $\sin(62°)$ **a)** $\tan(89°)$ **b)** $csc(40°)$

Radians to Degrees

3) $cos\left(\frac{\pi}{4}\right)$ **4)** $\csc\left(\frac{2\pi}{7}\right)$ **c)** $cot\left(\frac{5\pi}{12}\right)$

Unit 3 – Trig Identities to Remember						
Reciprocal		**Pythagorean**	**Cofunctions**		**Ratio Identities**	
$\boldsymbol{sin(\theta)} =$		**Main Pythagorean**	$\boldsymbol{sin(\theta)} =$		$\boldsymbol{tan(\theta)} =$	
$\boldsymbol{cos(\theta)} =$			$\boldsymbol{cos(\theta)} =$			
$\boldsymbol{tan(\theta)} =$		**Tangent Corollary**	$\boldsymbol{tan(\theta)} =$			
$\boldsymbol{csc(\theta)} =$			$\boldsymbol{csc(\theta)} =$		$\boldsymbol{cot(\theta)} =$	
$\boldsymbol{sec(\theta)} =$		**Cotangent Corollary**	$\boldsymbol{sec(\theta)} =$			
$\boldsymbol{cot(\theta)} =$			$\boldsymbol{cot(\theta)} =$			

<u>Simplify each of the following by using a Cofunction and/or a trig identity.</u>

5) $cos^2(10°) + cos^2(80°)$

6) $sin^2\left(\frac{\pi}{3}\right) + sin^2\left(\frac{\pi}{6}\right)$

7) $17cos^2(70°) + 17cos^2(20°)$

d) $16tan^2\left(\frac{\pi}{7}\right) + 16tan^2\left(\frac{5\pi}{14}\right)$

e) Make up an expression of your own using $sin^2(\theta)$, whose value is 113.

8) $\sin(70°)\big(csc(70°)\big)$

9) $12sec\left(\frac{\pi}{3}\right)cos\left(\frac{\pi}{3}\right)$

10) $15cot(34°)tan(56°)$

11) $cos^2(47°) + sin^2(13°) + cos^2(43°) + sin^2(77°)$

f) $3cos^2\left(\frac{\pi}{8}\right) + 4sin^2\left(\frac{\pi}{10}\right) + 4sin^2\left(\frac{2\pi}{5}\right) + 3cos^2\left(\frac{3\pi}{8}\right)$

Day 8 – Section 4.4 – Trig Values at any Angle

Objectives: Find Cofunctions and use Basic Trig Identities

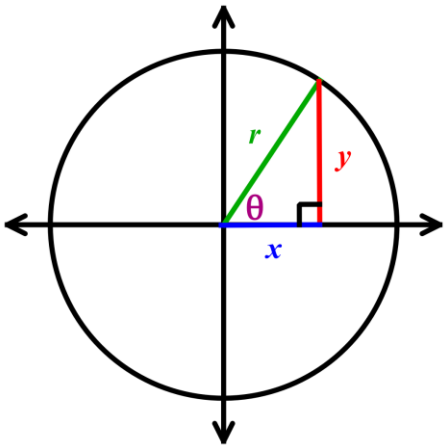
Review Questions of the day:

1) What is the formula for arc length?

2) What is $sin(0)$?

3) If $cos(t) = 3/5$ then find $sin(t)$ if t is in QUAD IV.

Recall the ratios for $sin(A)$, $cos(A)$, and $tan(A)$ in a circle of **radius r**, where A is in radians or degrees:



cosA =	x/r	sinA =	y/r	tanA =	y/x
secA =	r/x	cscA =	r/y	cotA =	x/y

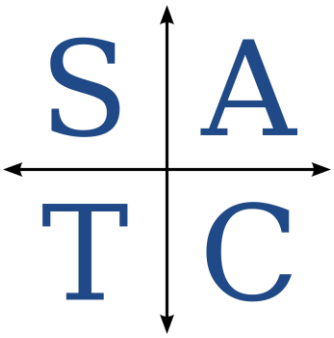
First, let's work on the signs in each quadrant. Given each situation, tell in which quadrant angle A terminates or lies.

1) $cos(A) > 0$ and $sin(A) > 0$

2) $sin(A) < 0$ and $cos(A) > 0$

3) $cos(A) < 0$ and $sin(A) < 0$

4) $tan(A) >$ and $cos(A) < 0$

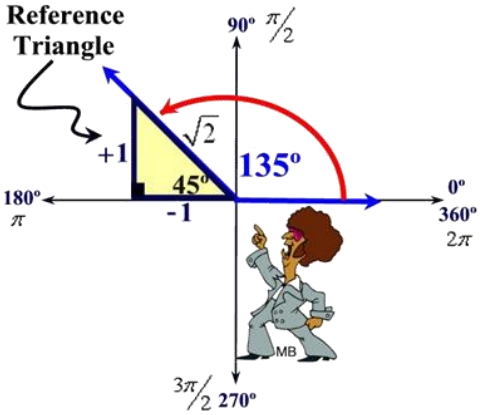


a) $csc(A) > 0$ and $tan(A) < 0$

b) $sec(A) < 0$ and $cot(A) > 0$

Once you know your basic trig values then you will know all other values for those functions with coterminal angles. Just remember to check your quadrant as well. Get a coterminal angle within one revolution and then use the **reference** angle to find the value.

Reference Angle:



<u>State the reference angle for each and then find the exact value for each.</u>

5) $cos\left(\frac{9\pi}{4}\right)$

6) $\sin\left(\frac{15\pi}{4}\right)$

7) $sin\left(-\frac{13\pi}{6}\right)$

8) $cot\left(\frac{19\pi}{6}\right)$

9) $cos\left(\frac{23\pi}{3}\right)$

10) $tan\left(\frac{7\pi}{3}\right)$

c) $tan\left(-\frac{11\pi}{4}\right)$

d) $csc\left(\frac{17\pi}{6}\right)$

e) $sin\left(\frac{11\pi}{3}\right)$

11) $sin(780°)$

f) $\cos(-1020°)$

For Quadrantals, there is no reference angle…you just figure out where the angle has landed. EAST, WEST, NORTH, or SOUTH

12) $tan\left(\frac{7\pi}{2}\right)$

13) $cot\left(\frac{9\pi}{2}\right)$

14) $sin(31\pi)$

15) $cos(14\pi)$

g) $\cos(191\pi)$

h) $sin\left(\frac{81\pi}{2}\right)$

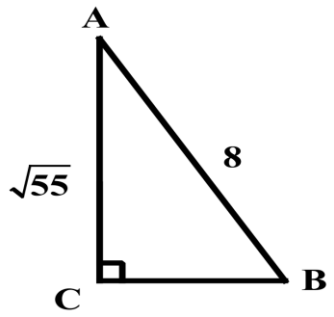
Day 9 – Section 4.4A – x, y, and r

Objectives: Find Cofunctions and use Basic Trig Identities

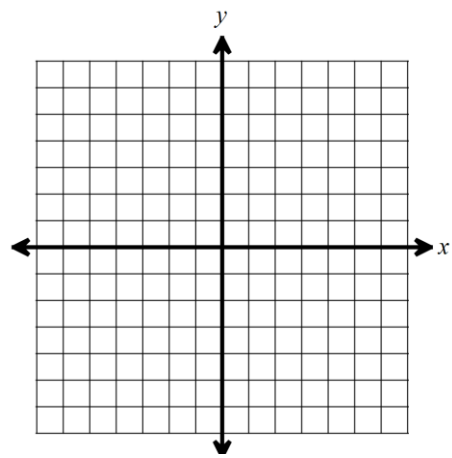
Review Questions of the day:

1) Find value of BC

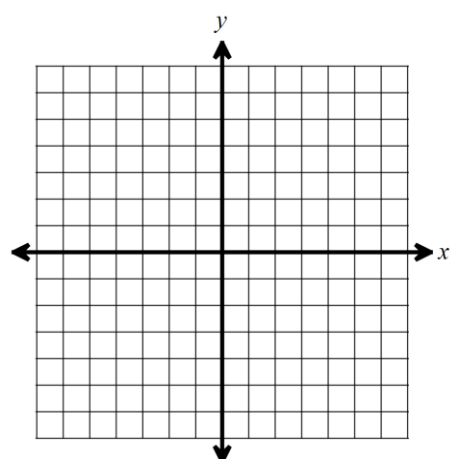
2) Find $m\angle B$



1) If $sin(A) = 3/5$ and A is in Quad I, find the $tan(A)$



2) If $sec(\theta) = 5/3$ and θ is in Quad IV, find $sin(\theta)$.



a) If $csc(B) = 5/12$ and B is in Quad II, find $sin(B)$.

3) If $(-5,-12)$ lies on the terminal side of an angle *A* then find all six trig ratios.

cosA	sinA	tanA
secA	cscA	cotA

b) If $(-2,3)$ lies on the terminal side of an angle *A* then find all six trig ratios.

cosA	sinA	tanA
secA	cscA	cotA

<u>Evaluate the following trig expressions.</u>

4) $sin\left(\frac{\pi}{2}\right)cos\left(\frac{\pi}{6}\right)+sin\left(\frac{\pi}{6}\right)cos\left(\frac{\pi}{2}\right)$

5) $sin\left(\frac{\pi}{3}\right)cos(0)+sin\left(\frac{2\pi}{3}\right)cos\left(\frac{\pi}{4}\right)$

c) $sin\left(-\frac{8\pi}{3}\right)tan\left(\frac{\pi}{4}\right)+cos\left(\frac{-7\pi}{6}\right)$

<u>Find the values of θ from $(0,2\pi]$ that satisfy each equation.</u>

6) $cos(\theta)=\frac{\sqrt{2}}{2}$

8. $sin(\theta)=-\frac{1}{2}$

9) $tan(\theta)=\frac{\sqrt{3}}{3}$

Fill in The Unit Circle