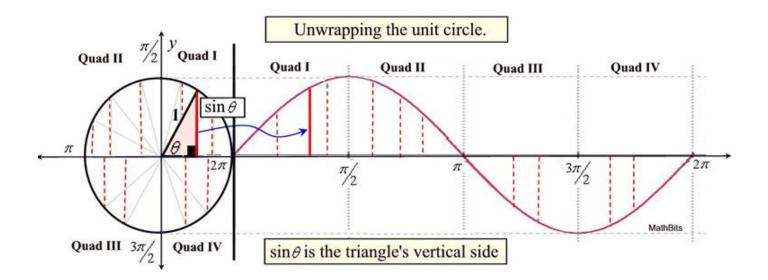
# **PreCalculus with TRIG – Unit 4 – Graphing Trig Functions**

## Day 4.5 - Intro to Graphing Sine and Cosine

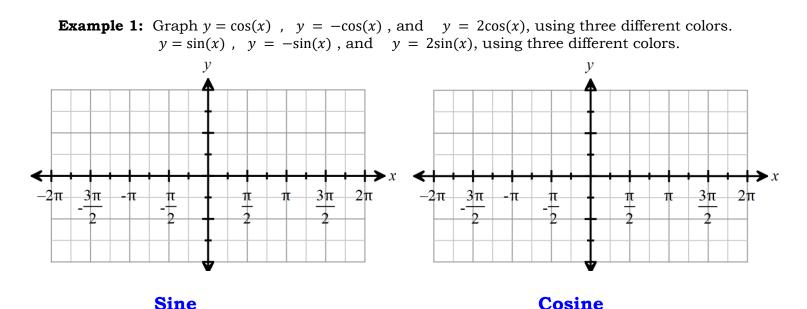
**Objectives:** SWBAT graph Sine and cosine functions Today, we begin graphing sin(x) and cos(x) in the x - y plane. The graphs of sin(x) and cos(x) are **periodic** with an **oscillating pattern**. We call these **sinusoidal** waves.

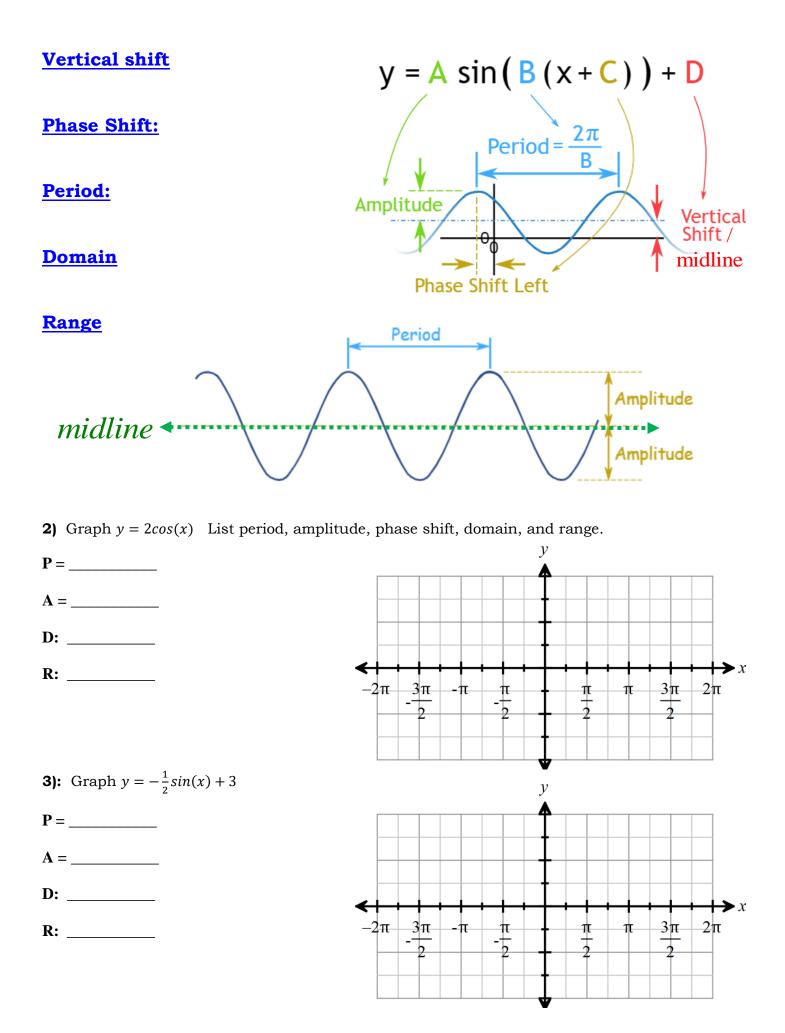


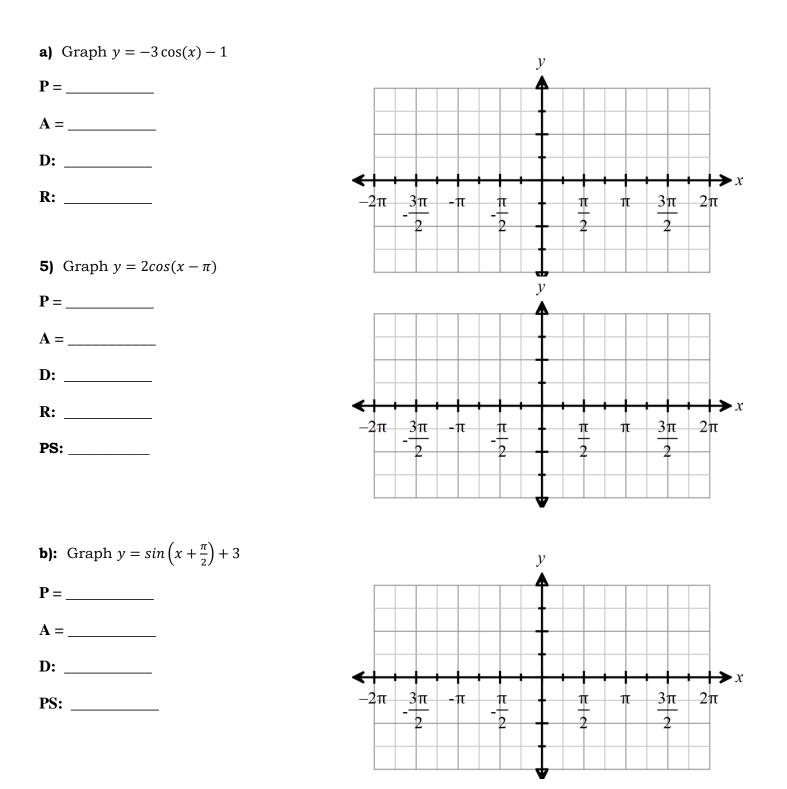
**Music** is composed of waves of different **frequencies and amplitudes** and these can be described using sin(x) / cos(x) waves. In fact most anything involving sound waves will rely on sin(x) / cos(x).

**GPS and cellphones** rely on triangulation and formulas involving sin(x) / cos(x)

**Signal transmission**, such as TV and radio broadcasting, involves sin(x) / cos(x) waves.







#### State the phase shift and vertical shift of each.

6) 
$$y = -5\cos\left(x + \frac{\pi}{4}\right) + 3$$
 7)  $y = -5\sin\left(x + \frac{\pi}{7}\right) - 19$  c)  $y = 3\cos(x - 3\pi) + 11$ 

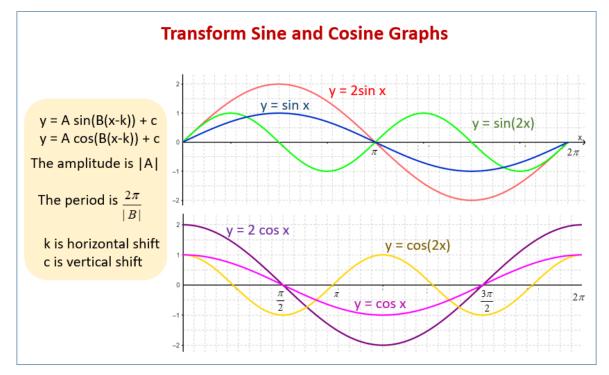
# **Day 4.5A – Graphing** sin(x) & cos(x) with Compressing and Stretching

**Objectives:** SWBAT graph sin(x) and cos(x) waves with periods other than  $2\pi$  as well as prepare

to incorporate phase shift into the graph

#### **Review Questions of the day:**

- 1) State the amplitude, period, and phase shift of  $y = -2\cos\left(x + \frac{\pi}{4}\right)$ .
- 2) State the domain and range of the above function.
- **3)** What effect does the 4 have on the graph of  $y = (x + 3)^2 + 4$ ?



GAP Amount = \_\_\_\_\_ This will help you find the max, min, and zeros quickly. To find the GAP amount, always divide the period by \_\_\_\_\_\_ for sin(x) and cos(x). Look at the base graph.

From this example, we conclude the following:

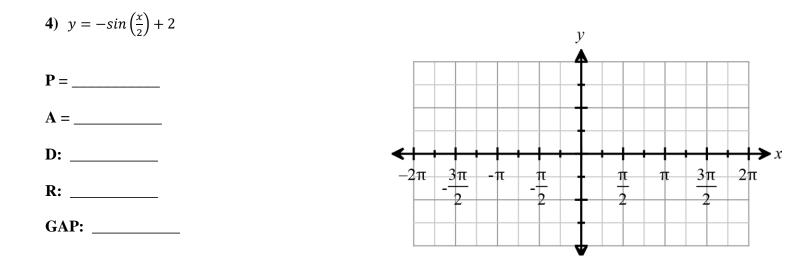
- 1) The period of  $y = A \cdot cos(Bx)$  is always \_\_\_\_\_. The value of B represents the number of waves completed in a normal period of  $2\pi$
- 2) The period of  $y = A \cdot sin(Bx)$  is always \_\_\_\_\_. The value of B represents the number of waves completed in a normal period of  $2\pi$

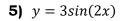
- **5)** When the value of **B** is **1**
- 6) When B is inside the parenthesis you should always it out first.

## Find the period of each of the following, then state if it is a stretch or shrink.

**1)** 
$$y = 2sin(0.5x)$$
 **2)**  $y = -4cos(5x)$  **a)**  $y = -3cos(\frac{x}{3})$ 

**3)** 
$$y = -3sin(\pi x) + 7$$
 **b)**  $y = 55sin(\frac{\pi}{3}x) - 5$ 

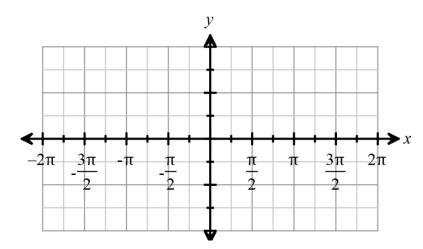


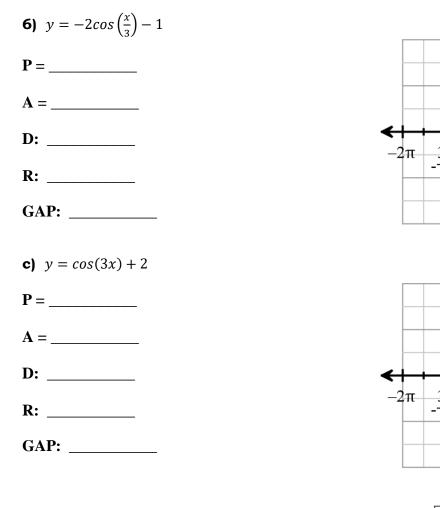


- P = \_\_\_\_\_
- A = \_\_\_\_\_
- D: \_\_\_\_\_

R: \_\_\_\_\_

GAP: \_\_\_\_\_





**7)**  $y = -2cos(-0.5\pi x)$ 

P = \_\_\_\_\_

A = \_\_\_\_\_

D: \_\_\_\_\_

R: \_\_\_\_\_

GAP: \_\_\_\_\_

**7)**  $y = -2\sin(\frac{\pi}{4}x)$ 

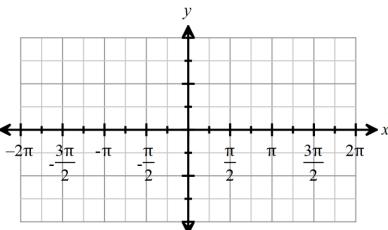
P = \_\_\_\_\_

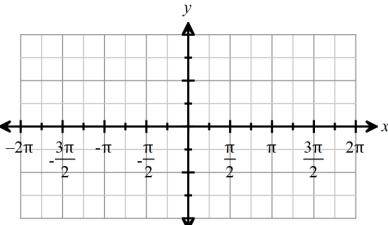
A = \_\_\_\_\_

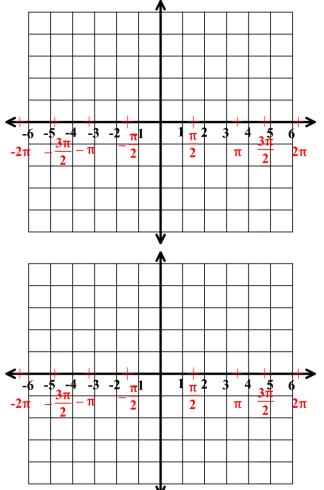
D: \_\_\_\_\_

R: \_\_\_\_\_

GAP: \_\_\_\_\_







# **Day 4.5C – Graphing** sin(x) & cos(x) with All Transformations

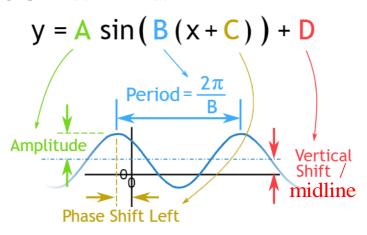
**Objectives:** SWBAT graph sin(x) and cos(x) with all transformations

#### **Review Questions of the day:**

**1)** Find the period of y = 2sin(8x).

**2)** Find sec(0).

**3)** What is the cofunction of *csc*(6)?



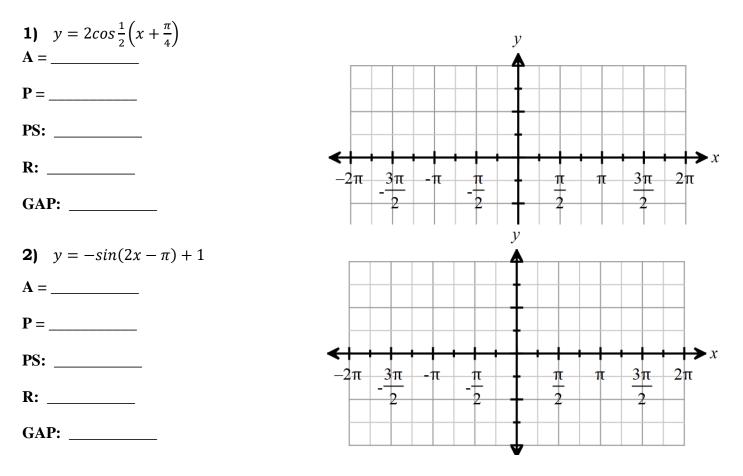
#### Follow these simple steps and you will be a GRAPHER!

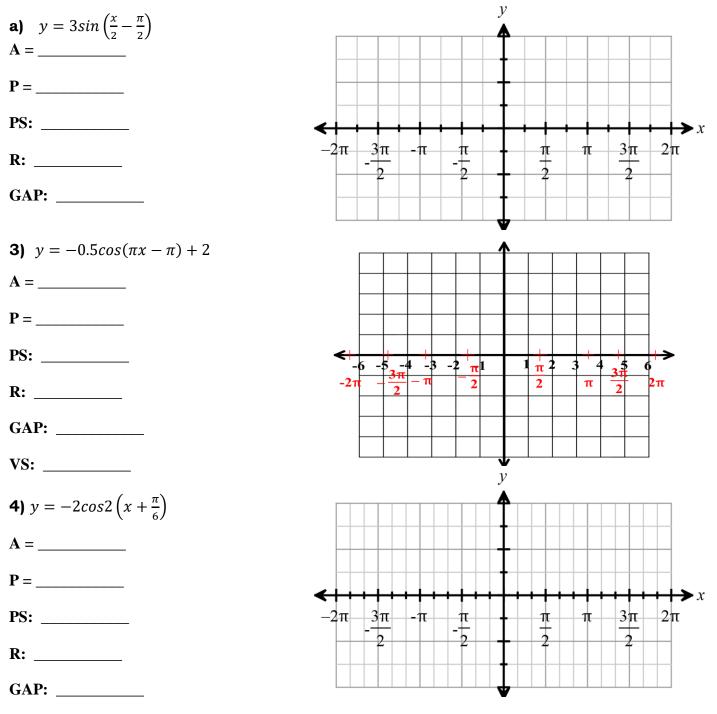
#### 1) Start at the phase shift and show your midline.

- **2)** Begin at a max or min for cos(x) or at the midline for sin(x).
- 3) Check your direction (positive / negative)
- 4) "Go" the GAP amount to get to the next "important" point on the graph.

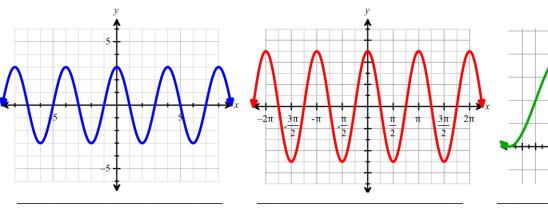
$$GAP = \frac{Period}{4}$$

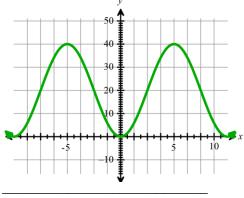
5) Graph each on the trig grid





#### Write 2 equations for the following graphs:





# Day 4.8 - Simple Harmonic Motion

**Objectives:** SWBAT graph sin(x) and cos(x) with all transformations

#### **Review Questions of the day:**

- 1) Find a coterminal angle for 11,345 degrees.
- **2)** Find  $cos\left(\frac{7\pi}{3}\right)$
- **3)** Find the range of  $y = 4\cos(x) + 2$

**SIMPLE HARMONIC MOTION:** any motion that follows an up and down oscillating pattern:

**Examples Include:** radio waves, TV waves, the motion of a vibrating guitar string, an object that bobs up and down, for example a spring or a buoy, basically anything that follows the sine or cosine wave.

**Basic Equations:**  $d = A \cdot cos(B)t$  or  $d = A \cdot sin(B)t$  A = Amplitude d = Distance or Displacement t = time  $Period = \frac{2\pi}{B}$  where B > 0. The period represents the time it takes for the motion to go through one complete cycle.  $Frequency = \frac{1}{Period}$  The frequency represents the number of complete cycles per unit of time.

When the object is at rest at t = 0, use sine

When the object is at a max or min at t = 0, use cosine.

**1)** Find an equation that represents the position of a ball attached to a spring hung from a ceiling. It is pulled down 7 inches and then released. If we ignore friction, the ball will continue oscillating on the end of the spring, and has a period of 8 seconds. The rest position for this ball is called the equilibrium position, d = 0 before you pull it down.



#### For Examples 2 – a find each of the following:

- a) the maximum displacement (amplitude)
- **b)** the frequency
- c) the time required for one cycle
- **d)** its distance at time = 0

**2)** 
$$d = 8cos(\pi)t$$
 **3)**  $d = 12sin(\frac{\pi}{4})(t)$  **a)**  $d = -9cos2t$ 

4) An object is attached to a coiled spring. It is pulled down and then released. The distance from the rest position at time 0 is 10 cm. The amplitude is 10 cm and the period is 6 seconds. Write an equation for the distance of the object from its rest position after t seconds.

**b)** A buoy is at rest at time 0. Then it begins to bob up and down, with a maximum displacement of 11 inches. The time to complete one cycle is 1.5 seconds. Write an equation for the simple harmonic motion of the buoy, assuming at time 0 that the buoy is on its way down from equilibrium.

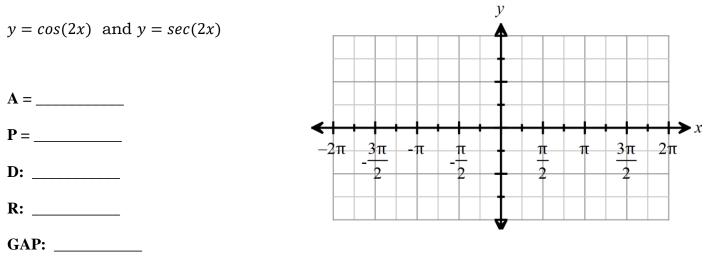
# **Day 4.6 – Graphing** csc(x) & sec(x)

**Objectives:** SWBAT graph csc(x) and sec(x)

#### <u>Review Questions of the day:</u>

- 1) State the period and phase shift of  $y = -2cos2(x + 45^{\circ})$ .
- 2) State the domain and range of the above function.
- **3)** Describe the transformation of the parabola  $y = (2x + 3)^2 + 2??$
- 4) How will y = cos(3x) compare with y = cos(x)?

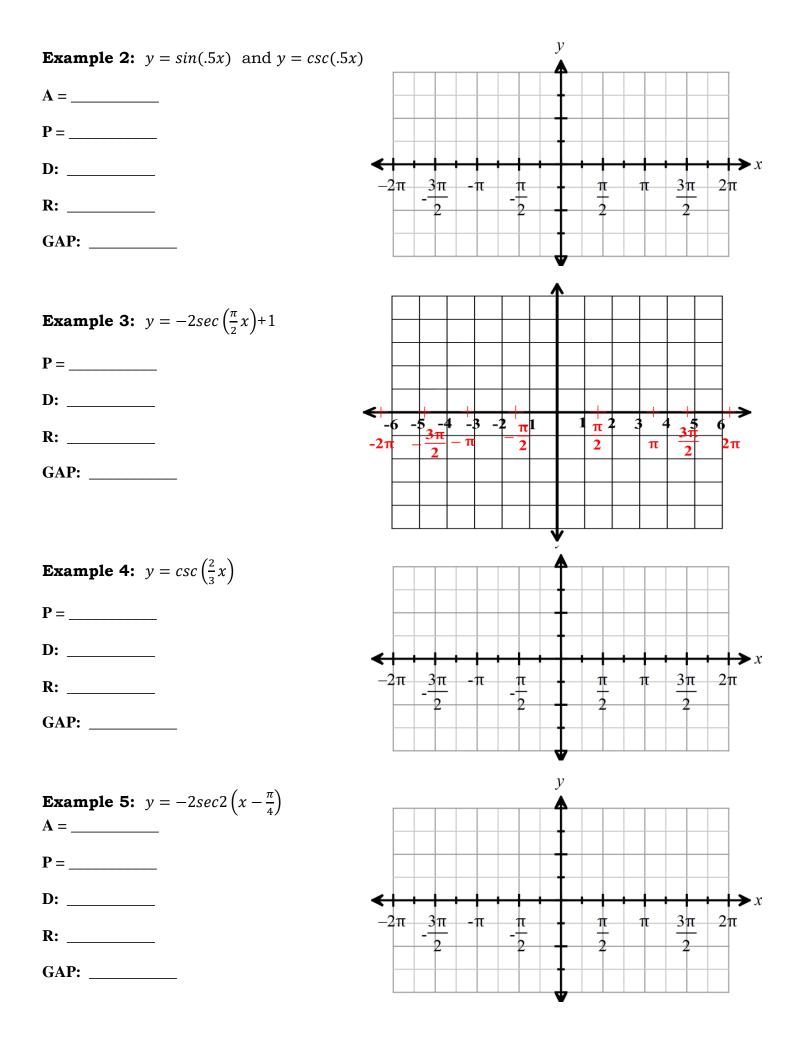
#### Graph each on the trig grid and state the amplitude, period, domain, and range.

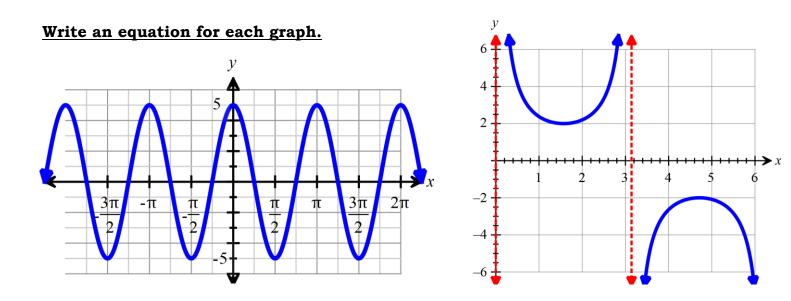


The **GAP amount** will help you find the max, min, and zeros quickly. Therefore, this helps you find the vertices of the parabolas and the vertical asymptotes. To find the GAP amount, always divide the **period by** \_\_\_\_\_\_ for sec(x) and csc(x). From this example, we conclude the following:

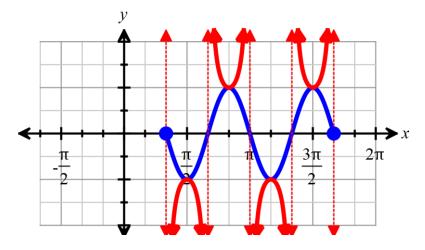
- The period of y = Asec(Bx C) is always \_\_\_\_\_. The value of B represents the number of waves completed in a normal period of 2π. The phase shift is \_\_\_\_\_\_. Graph cos(x) first and touch and flip.
- 2) The period of y = Acsc(Bx C) is always \_\_\_\_\_. The value of B represents the number of waves completed in a normal period of 2π. The phase shift is \_\_\_\_\_\_. Graph sin(x) first and touch and flip.

3) B > 1 \_\_\_\_\_ 0 < B < 1 \_\_\_\_\_





# What is the graph of the blue and red lines below?



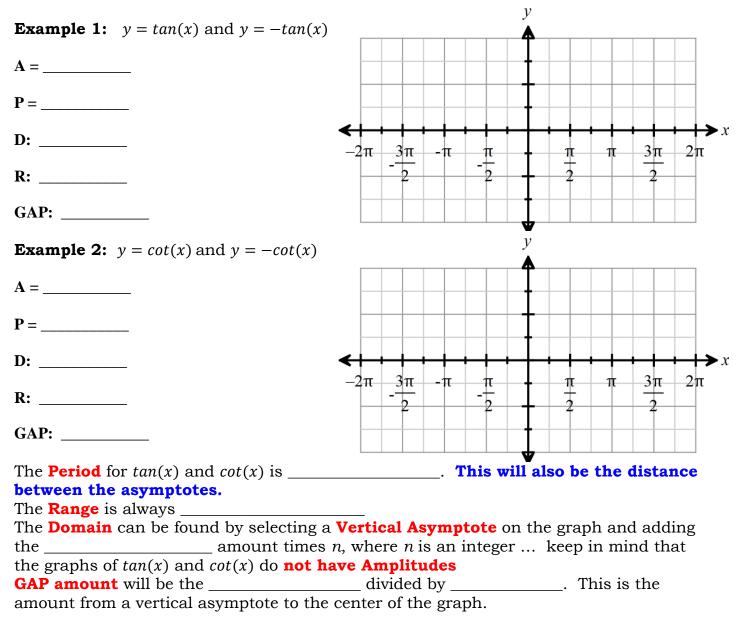
# **Day 4.6A – Graphing** tan(x) & cot(x)

**Objectives:** SWBAT Graph cot(x) and tan(x) base graphs as well as graphs with phase shifts

#### **Review Questions of the day:**

- **1.** Find the amplitude, period, and phase shift of  $y = -2cos^2(x \pi)$
- **2.** What is  $csc\left(\frac{\pi}{4}\right)$ ?
- **3.** If  $cos(x) = \frac{3}{5}$  and x is in Quadrant IV, find sin(x).

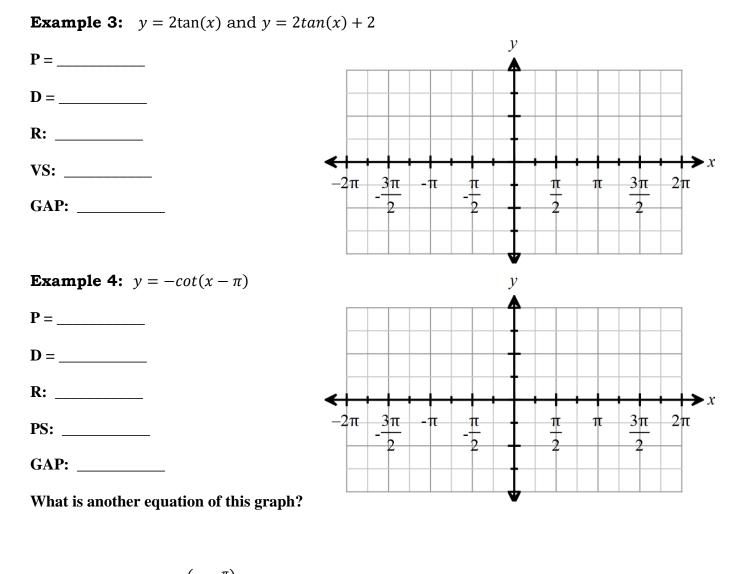
#### Graph the following using 2 different colors.



### When applying your Phase Shift.... know the patterns:

$$y = tan(x)$$
  $y = cot(x)$ 

## Graph the following using 2 different colors.



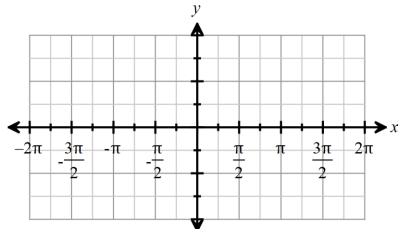
**Example 5:** 
$$y = tan(x + \frac{\pi}{4})$$
  
**P** = \_\_\_\_\_

D = \_\_\_\_\_

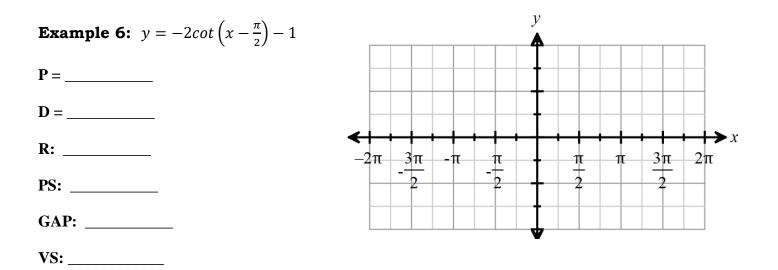
R: \_\_\_\_\_

PS: \_\_\_\_\_

GAP: \_\_\_\_\_



What is another equation of this graph?

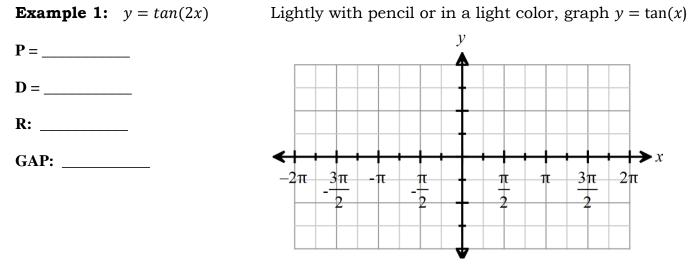


## **Day 4.6B – Graphing** tan(x) & cot(x) with Compressions and Stretches

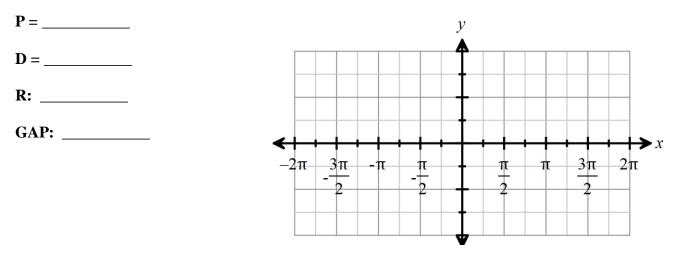
**Objectives:** SWBAT Graph cot(x) and tan(x) with compressions and stretches

#### <u>Review Questions of the day:</u>

- **1)** Find the amplitude, period, and phase shift of  $y = -2sin^3(x \pi)$
- **2)** What is  $sec\left(\frac{\pi}{4}\right)$ ?
- **3)** If  $cos(x) = \frac{12}{13}$  and x is in Quadrant IV, find sin(x).

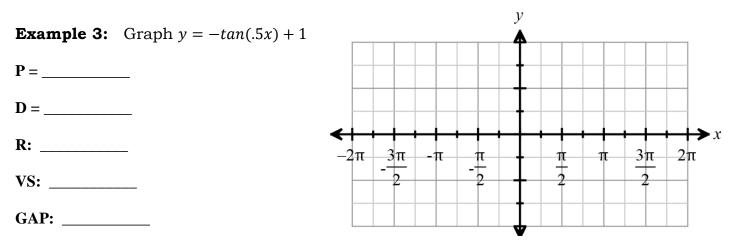


**Example 2:** Graph y = cot(3x) (Lightly with pencil or in a light color, graph y = cot(x)



#### **GUIDELINES:**

**Domain:**  $x \neq V.A. + period n$  where n is an integer Use this for both tan(x) and cot(x). GAP amount will be the \_\_\_\_\_\_ divided by \_\_\_\_\_\_. This is the amount from a vertical asymptote to the center of the graph. Use this for both tan(x) and cot(x).



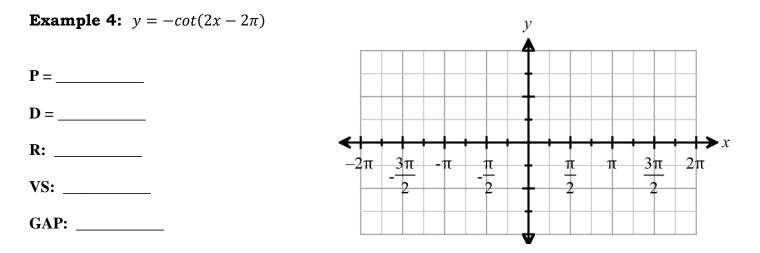
#### STEPS for Graphing when there is a phase shift and a compression or stretch

- Start at the **phase shift**
- Go the **GAP amount (period/2)** to get to a VA for tan(x) and to the "center" for cot(x)
- Draw the increasing or decreasing function...look at coefficient and function to decide

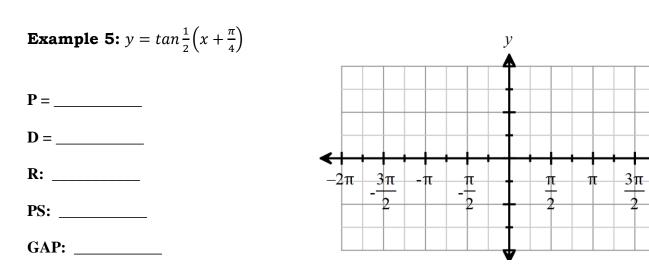
#### **REMEMBER THE PATTERNS AND SIMPLY MAKE THE TRANSFORMATIONS**

y = tan(x) (starts at center of snake then go to VA)

y = cot(x) (starts at VA then go to center of snake)

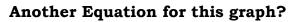


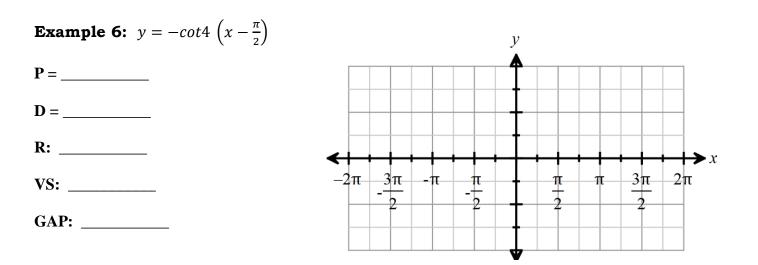
Another Equation for this graph?



 $\mapsto x$ 

2π





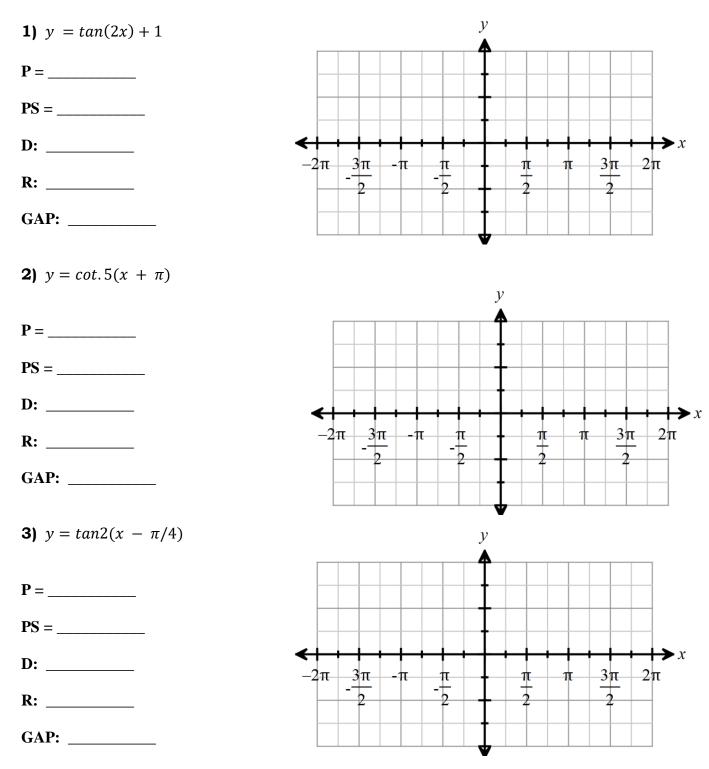
# **Day 4.6C – Graphing** tan(x) & cot(x) with all Transformations

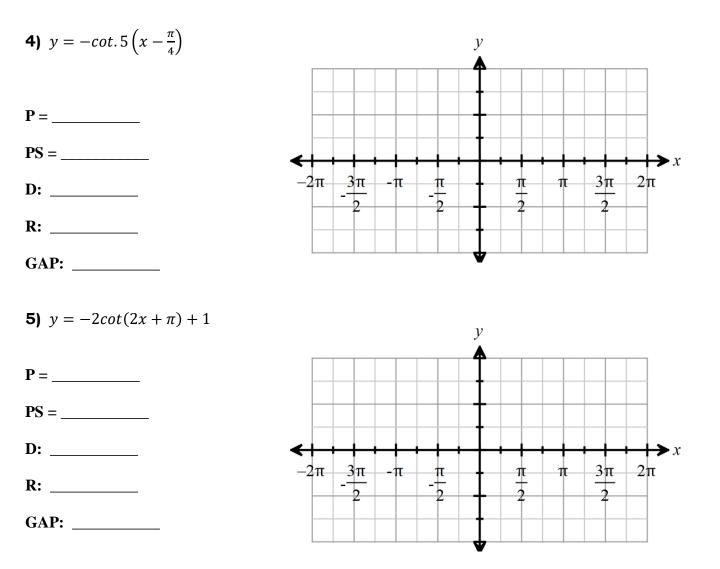
**Objectives:** SWBAT Graph cot(x) and tan(x) with all Transformations

#### **Review Questions of the day:**

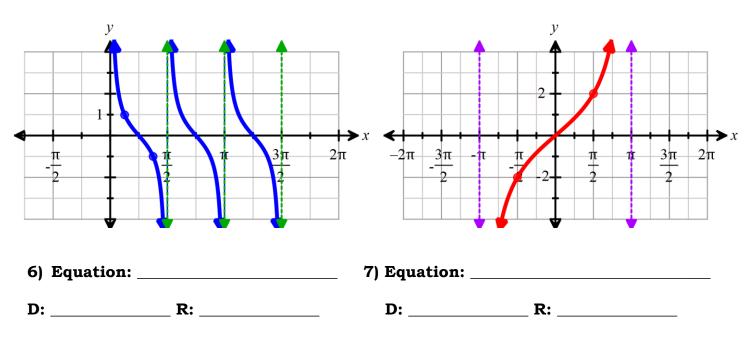
- **1)** What is the period of cos(3x)?
- **2)** What is the period of cot(4x)?

#### Graph each of the following and include the following information.

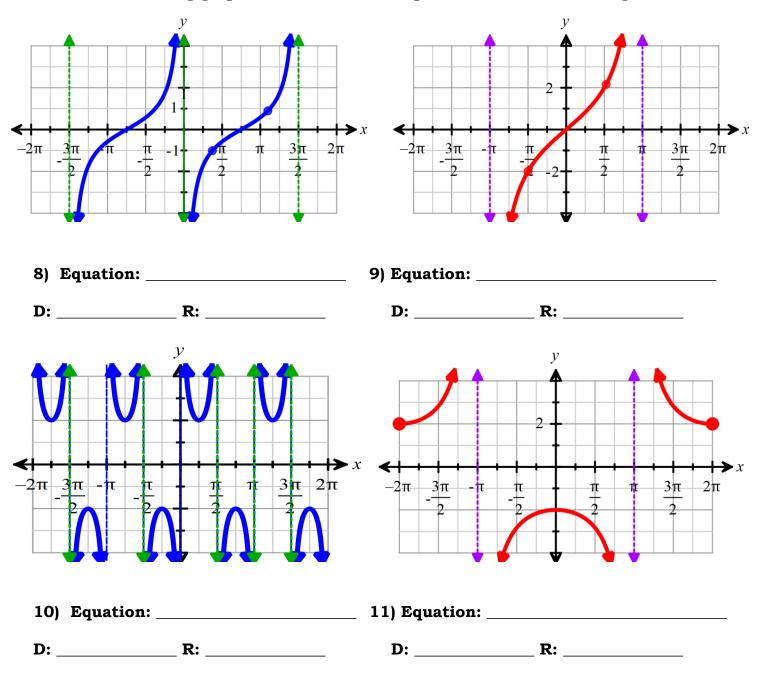




Given the following graphs below, write the equation, domain, and range for each.



## Given the following graphs below, write the equation, domain, and range for each.



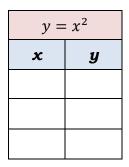
# **Day 4.7 – Inverses and Principal Values**

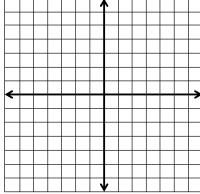
**Objectives:** SWBAT Find the value of inverse trig functions, both without and with a calculator. Sketch inverse trig functions. State the domain and range of inverse trig functions.

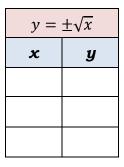
#### **Review Questions of the day:**

- **1)** What is the range for the graph of y = -7cos(x)?
- **2)** What is the range for the graph of y = 2csc(x)?
- **3)** If  $tan(A) = \frac{3}{7}$ , what are the possible values of angle A within [0, 360<sup>0</sup>)?
- 4) From Algebra, what are the 2 steps we take to find an inverse?

Let's use these to find the inverse of  $y = x^2$  and sketch their graphs in the same x - y plane.







#### Is this inverse a function?

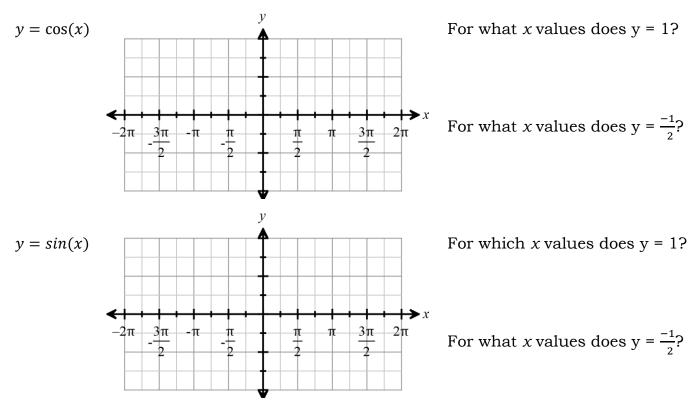
#### Vertical Line Test:

#### **Horizontal Line Test:**

Why does the calculator only give us 5 when we type in  $\sqrt{25}$  when  $(-5)^2$  also equals 25?

Trig Inverses work in the same exact way. An inverse of a trig function is NOT a function unless we restrict its domain. When we restrict their domain, the values are called \_\_\_\_\_\_ values.

Consider the base graphs for cos(x), sin(x), and tan(x). Sketch them here:



Discovering principal values for cosine...

Evaluate each by using your calculator. Get in degree mode so it is easier for you to recognize the angles.

**Ex.1**  $\cos^{-1}\left(\frac{1}{2}\right)$  **Ex. 2**  $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$  **Ex. 3**  $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 

**Ex.4** 
$$\cos^{-1}\left(-\frac{2}{3}\right)$$
 **Ex.5**  $\cos^{-1}\left(-\frac{3}{4}\right)$  **Ex. 6**  $\cos^{-1}\left(-\frac{7}{8}\right)$ 

So for cosines, the principal values are in Quadrants \_\_\_\_\_\_ or \_\_\_\_\_

## **PRINCIPAL VALUES FOR** $y = \cos(x)$

Quadrants \_\_\_\_\_ or \_\_\_\_\_.

#### Repeat for sine.

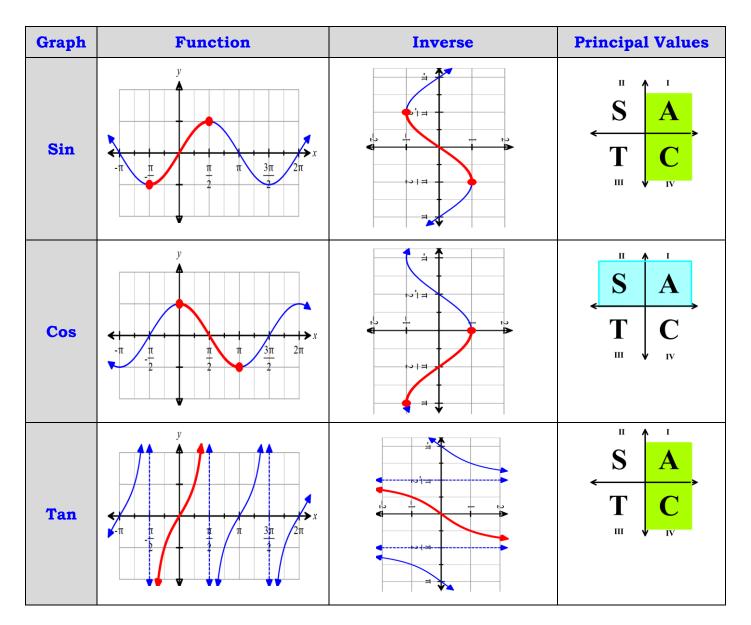
**Ex.7** 
$$sin^{-1}\left(\frac{1}{2}\right)$$
 **Ex. 8**  $sin^{-1}\left(\frac{\sqrt{2}}{2}\right)$  **Ex. 9**  $sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ 

**Ex.10** 
$$sin^{-1}\left(-\frac{2}{3}\right)$$
 **Ex.11**  $sin^{-1}\left(-\frac{3}{4}\right)$  **Ex. 12**  $sin^{-1}\left(-\frac{7}{8}\right)$ 

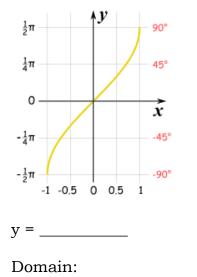
So for sine, the principal values are in Quadrants \_\_\_\_\_\_ or \_\_\_\_\_. PRINCIPAL VALUES FOR y = sin(x)[ , ] <u>Now investigate tangent.</u> Ex. 13  $tan^{-1}(1)$  Ex.14  $tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$  Ex.15  $tan^{-1}(-\sqrt{3})$  Ex. 16  $tan^{-1}(-2)$ 

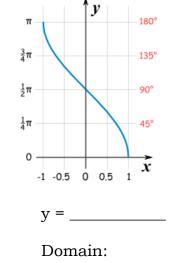
So for tangent, the principal values are in Quadrants \_\_\_\_\_\_ or \_\_\_\_\_\_. **PRINCIPAL VALUES FOR** y = tan(x)( , )

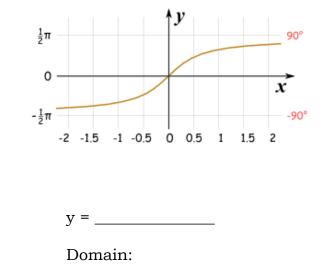
Why does the interval need to be open for tan(x)?



Label each one as  $y = sin^{-1}(x)$ ,  $y = cos^{-1}(x)$ , or  $y = tan^{-1}(x)$  and write the domain and range for each.







Range:

Range:

Range:

# Calculator Examples: Round degree measures to the nearest minute.

 $csc^{-1}(7)$   $sec^{-1}(3)$   $cot^{-1}(-2.4)$ 

Explain why **cos**<sup>-1</sup>(-9) doesn't work: \_\_\_\_\_

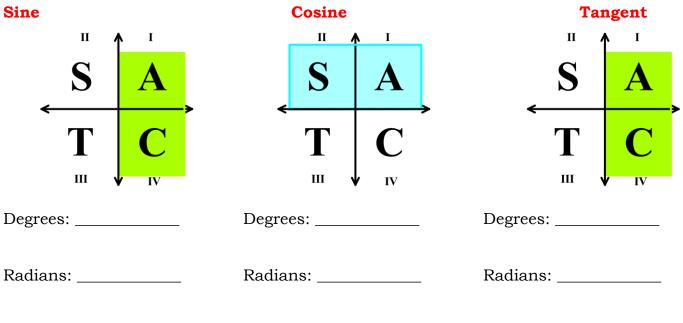
## **Day 4.7A – More Inverses and** x - y - r

**Objectives:** SWBAT use inverse trig functions to find principal values. Know the principal values for each trig function. Use Pythagorean Theorem to find specific trig ratios.

#### **Review Questions of the day:**

- **1)** When cot(x) = 0, what is cos(x)?
- **2)** When  $sin(x) = \frac{\sqrt{2}}{2}$ , then cos(x) =\_\_\_\_\_ or \_\_\_\_\_.
- **3)** When tan(x) = 1 then sin(x) =\_\_\_\_\_ or \_\_\_\_\_.

# **Principal Values**



## Find the value of each.

**Ex. 1**  $cos[sin^{-1}(1)]$ 

**Ex. 2**  $sin\left[cos^{-1}\left(\frac{\sqrt{2}}{2}\right)\right]$ 

**Ex. 3**  $tan[csc^{-1}(2)]$ 

**Ex. 4** 
$$cos\left[sin^{-1}\left(-\frac{1}{2}\right)\right]$$

**Ex.** 5  $cot[sec^{-1}(-1)]$ 

**Ex.** 6  $sin[sin^{-1}(-1)]$ 

**Ex. 7** 
$$cos\left[cos^{-1}\left(\frac{\pi}{6}\right)\right]$$
 Ex. 8  $cos^{-1}\left[cos\left(\frac{7\pi}{6}\right)\right]$ 

**Ex. 9** 
$$sin^{-1}\left[sin\left(\frac{-\pi}{4}\right)\right]$$
 **Ex. 10**  $sin^{-1}\left[sin\left(\frac{7\pi}{4}\right)\right]$ 

When will $cos^{-1}[cos(x)] = x$ ?	
When will $sin^{-1}[sin(x)] = x?$	-
When will $tan^{-1}[tan(x)] = x?$	

### Other way...

 $sin^{-1}[sin(y)] = \_ cos^{-1}[cos(y)] = \_ tan^{-1}[tan(y)] = \_$ 

# Find the exact value of each without using a calculator. Use $x^2 + y^2 =$

**Ex. 11**  $cos\left[sin^{-1}\left(\frac{4}{5}\right)\right]$  **Ex. 12**  $sin\left[cos^{-1}\left(\frac{-12}{13}\right)\right]$ 

**Ex. 13** 
$$tan[csc^{-1}(-3)]$$
 **Ex. 14**  $csc\left[sin^{-1}\left(\frac{-1}{5}\right)\right]$ 

## Check each of the above on your calculator.

# Day 4.7B – Inverses with SOH-CAH-TOA, and Domain & Range of Inverses

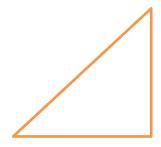
**Objectives:** SWBAT rewrite trig expressions using SOH-CAH-TOA and  $a^{2}+b^{2}=c^{2}$ . Discuss the domain and range of inverse trig graphs. Use inverse trig functions..

#### **Review Questions of the day:**

- **1)** Find  $\cos^{-1}\left(\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$
- **2)** Find  $cos[sin^{-1}(1)] + cos(0)$
- **3)** If tan(x) = -1 then sec(x) =\_\_\_\_\_ or \_\_\_\_\_.
- **4)** Explain what SOH–CAH–TOA means.

#### Write each expression as an algebraic expression of x. Assume x is positive.

**Ex. 1**  $sin[cos^{-1}(x)]$ 



**Ex. 2**  $csc\left[sin^{-1}\left(\frac{x}{3}\right)\right]$ 



**Ex. 3**  $sin[cos^{-1}(3x)]$ 



**Ex. 4**  $cos[tan^{-1}(x)]$ 

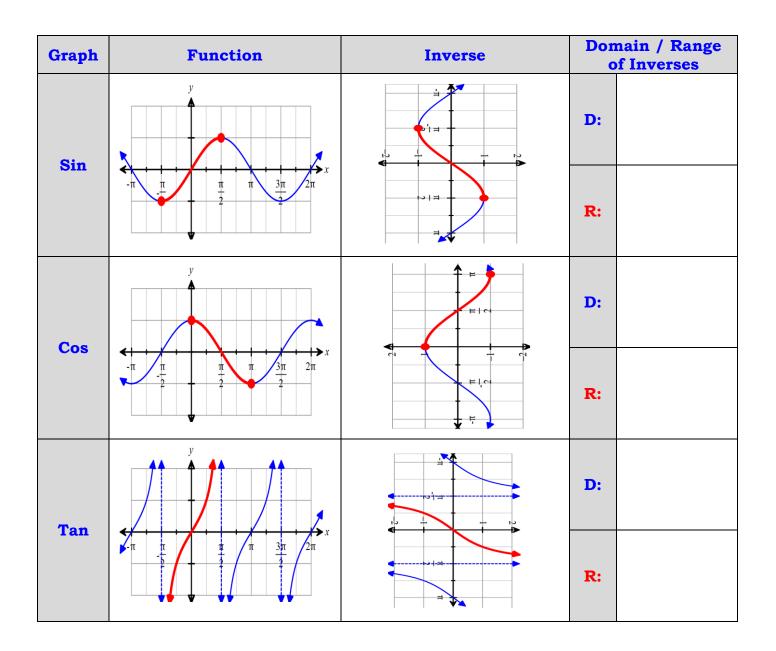


**Domain or Input:** 

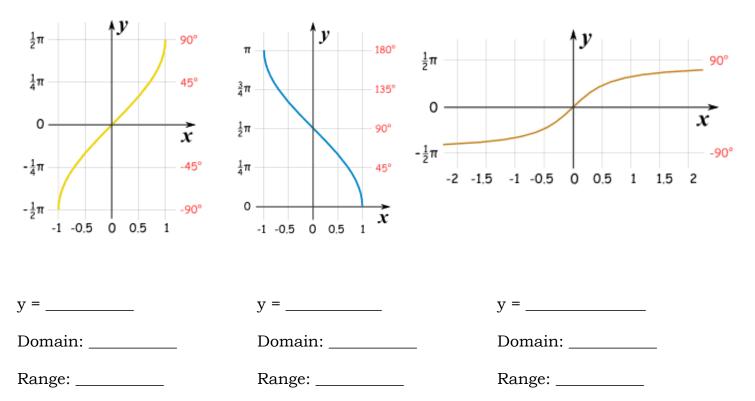
**Range or Output:** 

**Vertical Line Test:** 

Horizontal Line Test:



# Label each one as either $y = sin^{-1}(x)$ , $y = cos^{-1}(x)$ , or $y = tan^{-1}(x)$ . Then write the domain and range for each.



# When working with compositional trig functions, always start with the \_\_\_\_\_\_ and then work \_\_\_\_\_\_.

# Find the domain and range of each trig expression by using the above inverse graphs as a guide.

**Ex. 5**  $y = cos[sin^{-1}(x)]$  **Ex. 6**  $y = sin[cos^{-1}(x)]$  **Ex. 7**  $y = tan[tan^{-1}(x)]$ 

Domain:	Domain:	Domain:
Range:	Range:	Range:

# Simplify each trig expression. Do NOT use a calculator.

**Ex.** 8  $sin(0) + cos^{-1}(1) + 2 tan(0) - cos^{-1}\left(\frac{1}{2}\right)$ 

**Ex. 9**  $cos\left[tan^{-1}\left(\frac{8}{7}\right)\right]))$