PreCalculus with TRIG – Unit 5 – Trig Identities and Proofs

Day 1 – Section 5.1 – Trig Identities – Strategies Part I

Objectives: SWBAT Use trig identities in order to simply expressions and prove identities.

Review Questions of the day:

1) Find the phase shift of $y = 4cos\left(2x - \frac{\pi}{4}\right)$.

2) What is the minimum value of $y = 7sin(4x) - 9$?

3) What is $tan(x)$ in terms of $sin(x)$ and $cos(x)$?

4) Factor completely: $2x^4 - 512$

Review of Trig Identities from Unit 3							
Reciprocal		**Pythagorean**	**Cofunctions**		**Ratio Identities**	
$sin(\theta) =$		**Main Pythagorean**	$sin(\theta) =$		$tan(\theta) =$	
$cos(\theta) =$			$cos(\theta) =$			
$tan(\theta) =$		**Tangent Corollary**	$tan(\theta) =$			
$csc(\theta) =$			$csc(\theta) =$		$cot(\theta) =$	
$sec(\theta) =$		**Cotangent Corollary**	$sec(\theta) =$			
$cot(\theta) =$			$cot(\theta) =$			

Simplify each of the following expressions using trig identities.

1) $16cot(x)tan(x)$

2) $2csc(x)sin(x)cos(x)$

a) $\frac{7tan(x)}{cot(x)}$

3) $13sin^2(x) + 13cos^2(x)$

5) $csc(x)sin(x)cos(x)sec(x) + 9$

b) $\frac{sin^4(x)}{sin^2(x)}$

Trig Identities/ Proofs

An Identity is any equation that van be "proven" true. A trig identity involves at least one of the six trig functions. We can prove an identity by using several strategies.

STRATEGY 1: Change everything to $cos(x)$ and $sin(x)$.

6) $7\tan(x)sec(x)cot(x)cos(x)sin(x)csc(x) = 7$

c) $3tan(x)cos(x)cot(x)sec(x)sin^2(x)csc^2(x) = 3$

STRATEGY 2: Factor and simplify.

7) $sin^4(x) - cos^4(x) = sin^2(x) - cos^2(x)$

8) $\sec(x)\, cos^2(x) - \sec(x) = -\tan(x)\sin(x)$

STRATEGY 3: Split up one fraction.

9) $\frac{1+\cos(x)}{\sin(x)} = \csc(x) + \cot(x)$

d) $2\sec(x) + \tan(x) = \frac{2+\sin(x)}{\cos(x)}$

STARETEGY 4: Combining Fractions.

10) $\frac{\cos(x)}{\sin(x)} + \frac{\sin(x)}{\cos(x)} = \csc(x)\sec(x)$

e) $\cot(x) + \tan(x) = \frac{1}{\sin(x)\cos(x)}$

Day 2 – Section 5.1A – Trig Identities – Strategies Part II

Objectives: SWBAT Use trig identities in order to simply expressions and prove identities.

Review Questions of the day:

1) Simplify $114cos(x) + 114sin^2(x)$

2) State the domain of $y = tan(x)$.

3) Simplify $\frac{4}{3+\sqrt{2}}$

STRATEGY 5: Get the same denominator and combine terms as one fraction.

1) $\frac{\cos(x)}{1-\sin(x)} + \frac{1-\sin(x)}{\cos(x)} = 2\sec(x)$

a) $\cot(x) + \frac{\sin(x)}{1+\cos(x)} = \csc(x)$

Strategy 6: Use a conjugate. Multiply one side of the equation by conjugate/conjugate.

2) $\frac{\sec(x)+1}{\tan(x)} = \frac{\tan(x)}{\sec(x)-1}$

b) $\frac{1-\cos(x)}{\sin(x)} = \frac{\sin(x)}{1+\cos(x)}$

STRATEGY 7: Look for the other Pythagorean identities and make a substitution.

$tan^2(x) + 1 = sec^2(x)$ or $cot^2(x) + 1 = csc^2(x)$

3) $(tan^2(x) + 1)(cos^2(x) + 1) = tan^2(x) + 2$

Prove each identity by the method of your choice.

4) $1 - cos(x) = \frac{csc(x) - cot(x)}{csc(x)}$

5) $(\sec(x) - \tan(x))^2 = \frac{1-\sin(x)}{1+\sin(x)}$

Day 3 – Section 5.1B – Trig Identities – Apply All 7 Strategies

Objectives: SWBAT Use trig identities to simply expressions and prove identities.

Write down all 7 strategies to try for Trig Proofs

1. **Change everything to $cos(x)$ and $sin(x)$.**
2. **Factor and simplify.**
3. **Split up one fraction.**
4. **Combining Fractions.**
5. **Get the same denominator and combine terms as one fraction.**
6. **Use a conjugate. Multiply one side of the equation by conjugate/conjugate.**
7. **Look for the other Pythagorean identities and make a substitution.**

Verify Each Identity.

1) $1 - cscx = \frac{tanx - secx}{tanx}$

2) $tan^2x + 2 = sec^2x + 1$

3) $(cosx - sinx)^2 + (cosx + sinx)^2 = 2$

4) $\frac{cscx}{1 - cscx} = \frac{1}{sinx - 1}$

5) $tanx - cosx = tanxsinx + tanx - secx$

6) $\frac{sinx}{1-cotx}$ - $\frac{cosx}{tanx-1}$ $= sinx + cosx$

U Substitution

7) $cotx + \frac{sinx}{1+cosx}$ $= cscx$

8) $\frac{\tan(2\beta)+\cot(2\beta)}{\sec(2\beta)} = \csc(2\beta)$

9) $\frac{\tan(x)\cos(y)}{\sin(x)\cot(y)} = \frac{\sin(y)}{\cos(x)}$

Day 4 – Section 5.2 – Sum and Difference Formulas – Part I

Objectives: SWBAT Use sum and difference formulas for find exact values of Trig Functions

Review Questions of the day:

1) What is the period for $y = tan(x)$? **2)** What is $cot(2x)$ for $x = 0$? **3)** What is $cos2(\beta) + sin2(\beta)$?

Sum and Difference Formulas		
Cosine	**Sum**	
	Difference	
Sine	**Sum**	
	Difference	
Tangent	**Sum**	
	Difference	

Write the following in terms of $30°, 45°, 60°, or\ 90°$

1) $15°$ **2)** $105°$ **a)** $-15°$

Write the following in terms of $\frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, or\ \frac{\pi}{2}$

3) $\frac{\pi}{12}$ **4)** $\frac{7\pi}{12}$ **b)** $-\frac{\pi}{12}$

Find the exact value for each.

5) $cos(15°) = cos(60° - 45°)$

6) $sin(-15°) = sin(\qquad\qquad)$

c) $cos\left(\frac{7\pi}{12}\right) = cos(\qquad\qquad)$

8) $tan\left(-\frac{\pi}{12}\right) = tan(\qquad\qquad)$

10) $cos(40°)cos(20°) - sin\,(40°)sin(20°)$

11) $\frac{tan(20°)+tan(25°)}{1-tan(20°)tan(25°)}$

12) $tan\left(\frac{11\pi}{12}\right) = tan\left(\frac{2\pi}{3} + \qquad\qquad\right)$

Prove the following:

13) $cos\left(x - \frac{\pi}{2}\right) = sin(x)$

14) $tan(\pi - x) = -\tan(x)$

d) $sin(\alpha + \beta) + sin(\alpha - \beta) = 2sin(\alpha)\cos(\beta)$

15) $\frac{sin(\alpha+\beta)}{\cos(\alpha)\cos(\beta)} = \tan(\alpha) + \tan(\beta)$

Day 5 – Section – Section 5.2A – Sum and Difference Formulas – Adding Separate Trig Functions Together

Objectives: SWBAT Use sum and difference to evaluate expressions when given trig ratios, instead of angles.

Review Questions of the day:

1) $\frac{\frac{1}{2}}{\frac{3}{4}}$

2) $\frac{\frac{1}{2}}{\frac{3}{4}}$

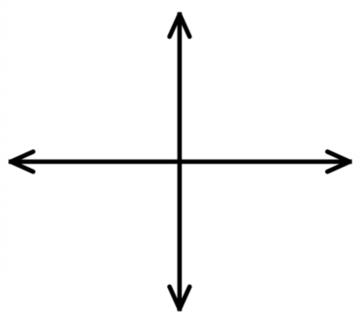
a) $\frac{-\frac{8}{3}}{-\frac{7}{9}}$

For 1 – 3, evaluate each of the following:

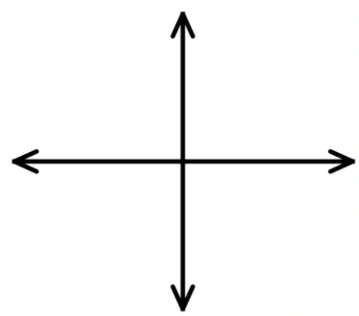
Given $sin(x) = \frac{12}{13}$ and $cos(y) = \frac{3}{5}$ with both x and y in Quadrant I.

$sin(x)$	$cos(x)$	$tan(x)$
$sin(y)$	$cos(y)$	$tan(y)$

x

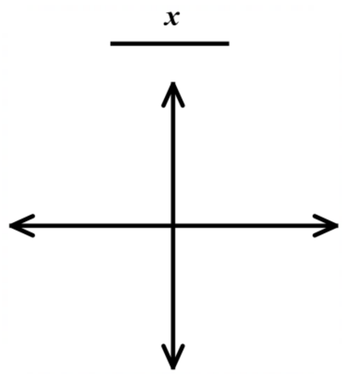


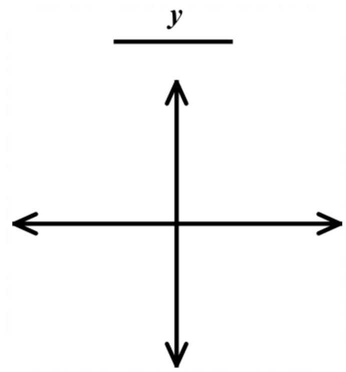
y



$sin(x+y)$	$cos(x+y)$	$tan(x+y)$

Given $sin(x) = -4/5$ and $sec(y) = 13/5$ with x in Quad III and y in Quad IV.		
$\boldsymbol{sin(x)}$	$\boldsymbol{cos(x)}$	$\boldsymbol{tan(x)}$
$\boldsymbol{sin(y)}$	$\boldsymbol{cos(y)}$	$\boldsymbol{tan(y)}$

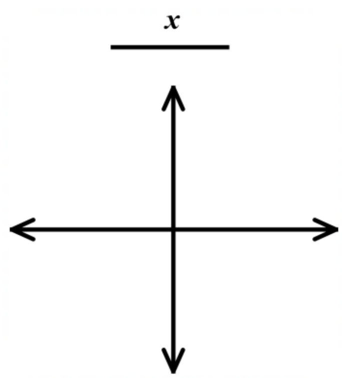


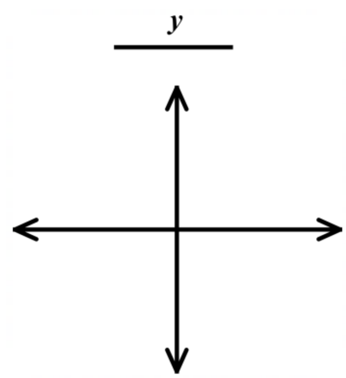


$\boldsymbol{sin(x+y)}$	$\boldsymbol{cos(x+y)}$	$\boldsymbol{tan(x+y)}$

You try….

Given $sin(x) = -4/5$ and $sec(y) = 13/5$ with x in Quad III and y in Quad IV.		
$\boldsymbol{sin(x)}$	$\boldsymbol{cos(x)}$	$\boldsymbol{tan(x)}$
$\boldsymbol{sin(y)}$	$\boldsymbol{cos(y)}$	$\boldsymbol{tan(y)}$





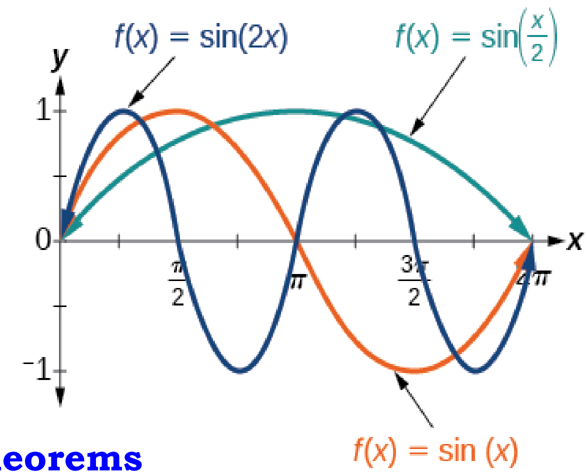
$\boldsymbol{sin(x+y)}$	$\boldsymbol{cos(x+y)}$	$\boldsymbol{tan(x+y)}$

Day 6 – Section – Section 5.3 – Double Angle Formulas

Objectives: SWBAT Use Double Angle Formulas

Double Angle

x* vs 2*x



Deriving Formulas from Trig Sum Theorems

$sin(2x) =$

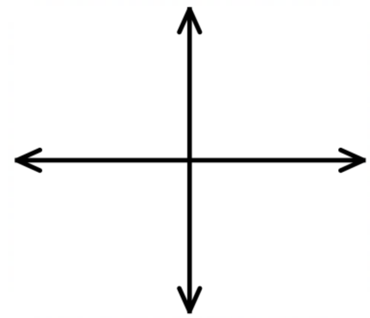
$tan(2x) =$

$cos(2x) =$

Sine	Tangent		Cosine
		1	
	Tangent Ratio Identity Rule	2	
		3	

If $sin(A) = 4/5$ and A is in Quadrant II, find

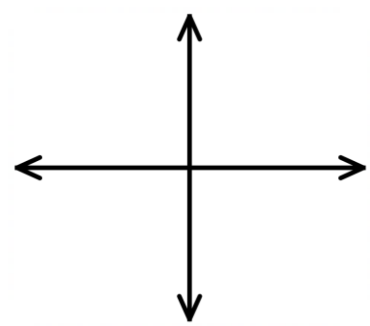
$sin(A)$	$cos(A)$	$tan(A)$



$sin(2A)$	$cos(2A)$	$tan(2A)$ Using Tangent Rule

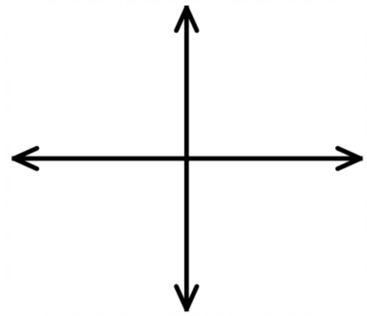
If $cot(\beta) = 5$ and β is in Quadrant III, find

$sin(\beta)$	$cos(\beta)$	$tan(\beta)$



$sin(2\beta)$	$cos(2\beta)$	$tan(2\beta)$ Using Ratio Identity

c) You Try….

<table>
<tr><td colspan="3">If $tan(\theta) = -4/7$ and θ is in Quadrant II, Find the following and check with your calculator.</td><td rowspan="3"></td></tr>
<tr><td>$\boldsymbol{sin}(\theta)$</td><td>$\boldsymbol{cos}(\theta)$</td><td>$\boldsymbol{tan}(\theta)$</td></tr>
<tr><td></td><td></td><td></td></tr>
</table>

$\boldsymbol{sin(2\theta)}$	$\boldsymbol{cos(2\theta)}$	$\boldsymbol{tan(2\theta)}$ **– Your choice in method**
	Calculator Check	

Rewrite each expression as the Double Angle of sine, cosine, or tangent (respectively) and then find the EXACT value.

2) $2sin(75)cos(75)$

3) $2sin(-15)cos(-15)$

4) $2cos^2\left(\frac{\pi}{12}\right) - 1$

b) $cos^2\left(\frac{\pi}{8}\right)sin^2\left(\frac{\pi}{8}\right)$

5) $\frac{2tan\left(-\frac{\pi}{12}\right)}{1-tan\left(-\frac{\pi}{12}\right)}$

6) $1 - 2sin^2(4x)$

Verify each Double Angle Identity.

7) $sin(2x) - tan(x) = tan(x)cos(2x)$

8) $(sin(x) + cos(x))^2 = 1 + sin(2x)$

9) $\cot(x) = \frac{\sin(2x)}{1-\cos(2x)}$

Day 7 – Section – Section 5.3A – Half Angle Theorems Introduction

Objectives: SWBAT Use Half Angle Theorems to evaluate expressions

Review Questions of the day:

Using Half-Angle Formulas:

Sine	Cosine	Tangent	
		1	
		2	
		3	

$\pm$ **Meaning**	
$\frac{\alpha}{2}$ **Meaning**	

Find the exact value of each by using a half angle formula.

1) $cos(15°) = cos\left(\frac{30°}{2}\right)$

Formula Used	Quadrant I will end in	Sign of the trig function in that Quadrant	Work
Calculator Check			

2) $sin(75) = sin\left(\underline{\hspace{3em}}\right)$

Formula	Quadrant I will end in	Sign of the trig function in that Quadrant	Work
Calculator Check			

a) $tan\left(\frac{\pi}{12}\right)$

Formula	Quadrant I will end in	Sign of the trig function in that Quadrant	Work
Calculator Check			

4) $cos\left(\frac{-3\pi}{8}\right)$

Formula	Quadrant I will end in	Sign of the trig function in that Quadrant	Work
Calculator Check			

Day 8 – Section – Section 5.3B – Half Angle Theorems – Part II

Objectives: SWBAT Use Half Angle Theorems to evaluate expressions when given trig ratios, instead of angles.

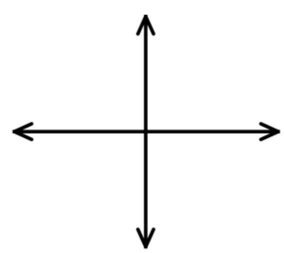
"No Angle Given" Problems

	x interval on Unit Circle	$\left(\frac{x}{2}\right)$ will be in interval	Then $\left(\frac{x}{2}\right)$ will exist in Quadrant....
Quad I			
Quad II			
Quad III			
Quad IV			

Therefore, if x is in Quadrant I or II then $\left(\frac{x}{2}\right)$ will be in __________________.

If x is in Quadrant III or IV then $\left(\frac{x}{2}\right)$ will be in ______________________.

This is true for all angles between 0 and 2π.

<table>
<tr><td colspan="3">Given $cos(A) = 12/13$ and A is in Quadrant I, find the following..</td><td rowspan="3"></td></tr>
<tr><td>$sin(A)$</td><td>$cos(A)$</td><td>$tan(A)$</td></tr>
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<table>
<tr><td>$sin\left(\frac{A}{2}\right)$</td><td>Is Half Angle Pos / Neg ?</td><td>$cos\left(\frac{A}{2}\right)$</td><td>Is Half Angle Pos / Neg ?</td><td>$tan\left(\frac{A}{2}\right)$</td><td>Is Half Angle Pos / Neg ?</td></tr>
<tr><td colspan="2" rowspan="2"></td><td colspan="2" rowspan="2"></td><td>Half Angle Rule</td><td></td></tr>
<tr><td>Ratio Identity</td><td></td></tr>
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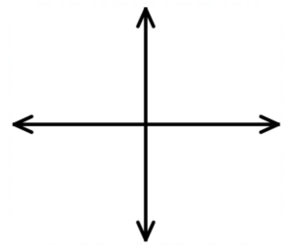
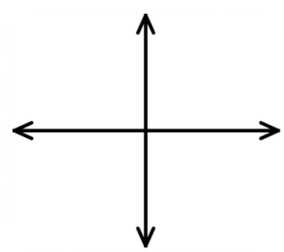
Given $sin(\theta) = -7/25$ and θ is in Quadrant III, find the following.			
$\boldsymbol{sin}(\theta)$	$\boldsymbol{cos}(\theta)$	$\boldsymbol{tan}(\theta)$	

$\boldsymbol{sin}\left(\frac{\theta}{2}\right)$	**Is Half Angle Pos / Neg ?**	$\boldsymbol{cos}\left(\frac{\theta}{2}\right)$	**Is Half Angle Pos / Neg ?**	$\boldsymbol{tan}\left(\frac{\theta}{2}\right)$	**Is Half Angle Pos / Neg ?**
				Half Angle Rule	
				Ratio Identity	

You Try…

Given that $cot(a) = -3$ and a is in Quadrant IV, find the following			
$\boldsymbol{sin}(\theta)$	$\boldsymbol{cos}(\theta)$	$\boldsymbol{tan}(\theta)$	

$\boldsymbol{sin}\left(\frac{\theta}{2}\right)$	**Is Half Angle Pos / Neg ?**	$\boldsymbol{cos}\left(\frac{\theta}{2}\right)$	**Is Half Angle Pos / Neg ?**	$\boldsymbol{tan}\left(\frac{\theta}{2}\right)$	**Is Half Angle Pos / Neg ?**
				Half Angle Rule	
				Ratio Identity	

Day 9 – Section – Section 5.3C – Power Reducing

Objectives: SWBAT Use Half Angle Theorems to evaluate expressions when given trig ratios, instead of angles.

In order to derive these formulas, use $\boldsymbol{cos(2x)}$ formulas and solve for $\boldsymbol{sin^2(x)}$ and $\boldsymbol{cos^2(x)}$**…**

$cos(2x) = 1 - 2sin^2(x)$ $\qquad$ $cos(2x) = 2cos^2(x) - 1$ $\qquad$ $\tan^2(x) = \frac{\boldsymbol{sin^2(x)}}{\boldsymbol{cos^2(x)}}$

Power Reducing Formulas		
Sine	**Cosine**	**Tangent**

Power Reducing Formulas are used in calculus to explore the rate of change over time. It is a way of rewriting a trig expression so that the power is smaller – therefore – reducing the power.

1) Write an equivalent expression for sin^2x that does not contain a power of a trig function greater than one.

Rewrite with Formula	Distribute	Rewrite as individual Fraction Terms	Simplify
Do you have any powers greater than one? Keep Going….			

2) sin^4x

Rewrite with Formula	Distribute	Rewrite as individual Fraction Terms	Simplify
Do you have any powers greater than one? Keep Going....			

3) $8cos^4(x)$ **HINT:** Pretend this is a $cos^4(x)$ and distribute the 8 at the end

Rewrite with Formula	Distribute	Rewrite as individual Fraction Terms	Simplify
Do you have any powers greater than one? Keep Going....			

3) $4sin^2(x)cos^2(x)$

a) $\frac{2}{3}sin^4(\theta)$

Trig Identities Wrap-Up Sheet

Reciprocal		Cofunctions		Even – Odd		Quotient		Pythagorean
$sin(\theta)$		$sin(\theta)$		$sin(-\theta)$		$tan(\theta)$		
$cos(\theta)$		$cos(\theta)$		$cos(-\theta)$				
$tan(\theta)$		$tan(\theta)$		$tan(-\theta)$				
$csc(\theta)$		$csc(\theta)$		$csc(-\theta)$		$cot(\theta)$		
$sec(\theta)$		$sec(\theta)$		$sec(-\theta)$				
$cot(\theta)$		$cot(\theta)$		$cot(-\theta)$				

Sine				
Sum	Difference	Double Angle	Half Angle	Power Reduce

Cosine					
Sum	Difference	Double Angle		Half Angle	Power Reduce
		1			
		2			
		3			

Tangent					
Sum	Difference	Double Angle	Half Angle		Power Reduce
			1		
			2		
			3		
In terms of sin/cos					