Precalculus with TRIG – Unit 8 – Polar

Day 1 – Section 6.3 – Introduction to Polar Coordinates

Objectives: SWBAT Plot polar points and develop the conversions between polar and rectangular coordinates

Review Questions of the Day:

1) What is the period for $y = sinx$? For $y = cotx$?

2) What is the expansion of $cos(x + y)$?

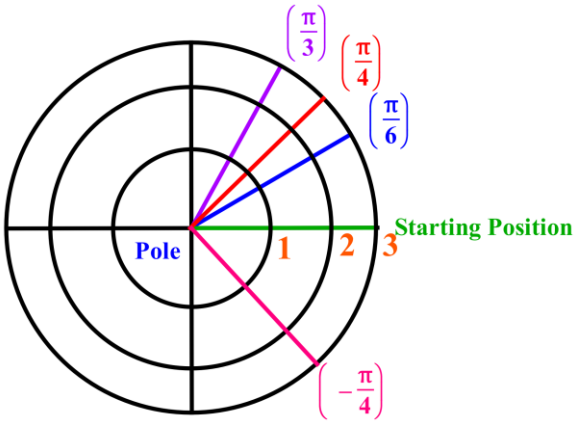
3) What needs to happen in order to have two triangles when given (SSA) SOAS?

Rectangular Coordinates:

Ordered pairs (x, y) tell us how far to go horizontally and vertically from the origin.

Polar Coordinates:

Ordered pairs are given in terms of a **radius** (distance from the origin) and an **angle** (measured in degrees or radians) from the positive *x*-axis. These pairs are written as (r, θ). The plane on which the points are graphed is called the **Polar Plane** and the origin is called the **pole**. The polar axis is the horizontal ray that extends to the right.



Graph each point on each polar grid/plane.

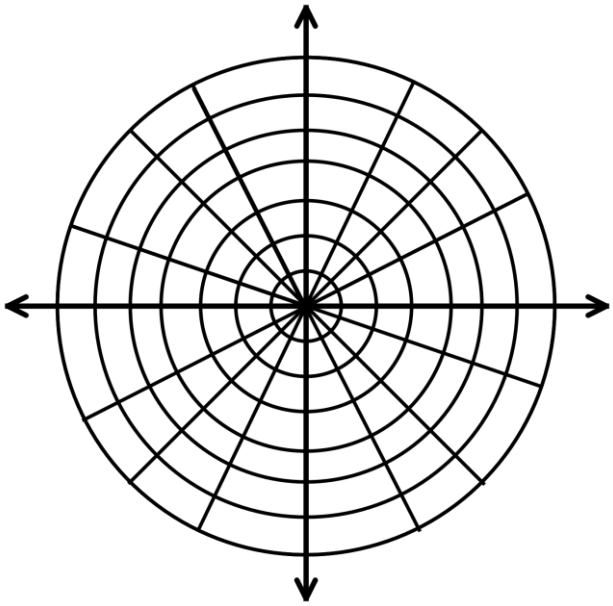
1) $A = \left(3, \frac{\pi}{6}\right)$

2) $B = \left(6, \frac{\pi}{4}\right)$

3) $C = \left(-2, \frac{\pi}{4}\right)$

a) $D = \left(-4, \frac{\pi}{4}\right)$

b) $E = \left(-5, \frac{\pi}{3}\right)$



4) Write the point $F = (0,7)$ (rectangular form) at least two different polar ways within one revolution

Given the point on the polar graph, write the 4 ways to write that point.

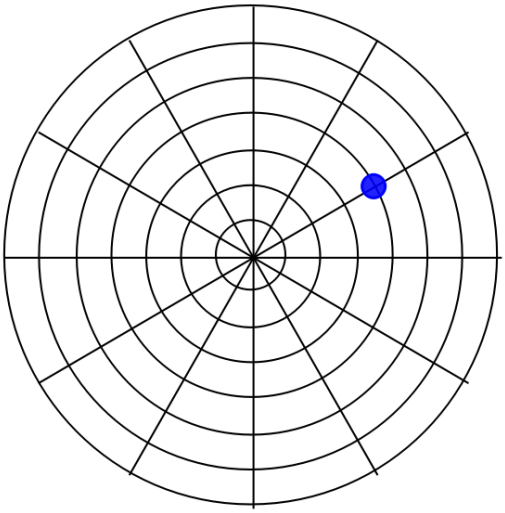
5)

_____

_____

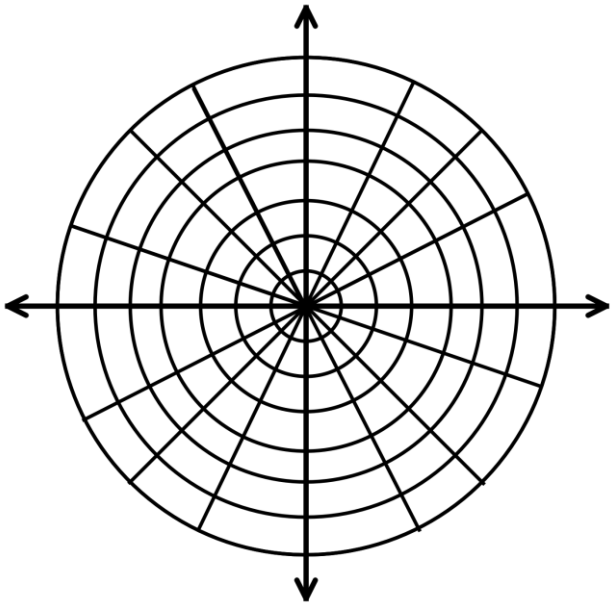
_____

_____



Graph each point on each polar grid/plane. Also tell three other ways to write the point within one revolution. Then give another point for which $r > 0$ and $2\pi < \theta < 4\pi$

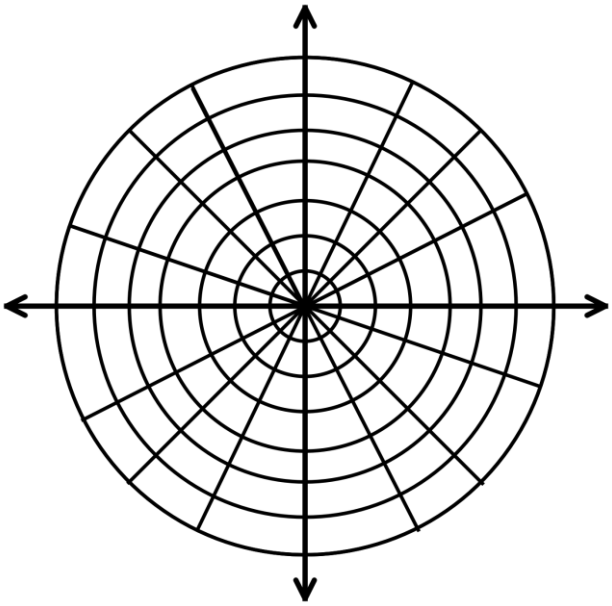
6) $\left(3, \frac{\pi}{6}\right)$



3 other ways: _____

Another point: _____

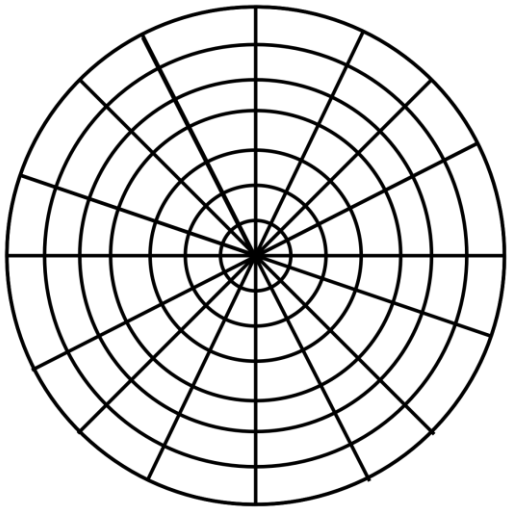
7) $\left(7, \frac{2\pi}{3}\right)$



3 other ways: _____

Another point: _____

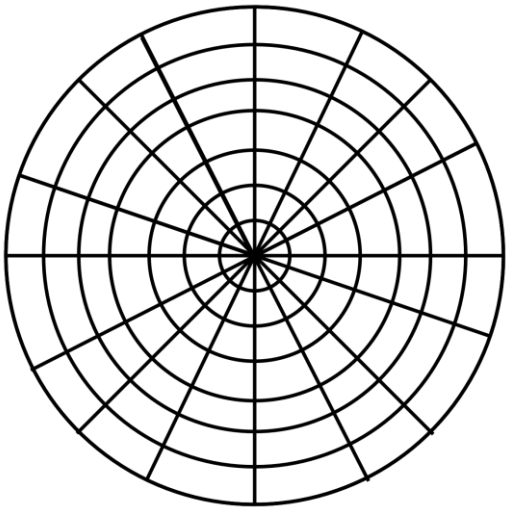
8) $(1, -30°)$



3 other ways: _____

Another point: _____

c) $\left(-6, \frac{-2\pi}{3}\right)$



3 other ways: _____

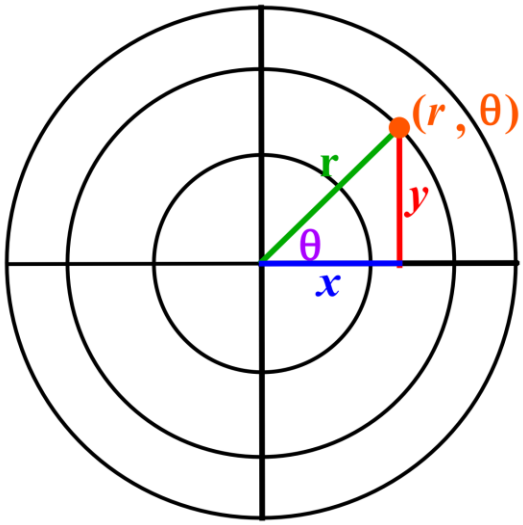
Another point: _____

Conversions:

Each point in rectangular form can be converted into polar form and vice versa.

Changing from Polar to Rectangular:

Changing from Rectangular to Polar:



$x =$ _____

$y =$ _____

$r =$ _____

$\theta =$ _____

Day 2 – Section 6.3A – Conversation of Polar Points

Objectives: SWBAT Convert from polar to rectangular coordinates and then rectangular to polar coordinates.

Review Questions of the Day:

1) Write the point $(4, 60°)$ three other ways within one revolution.

2) How many triangles if $A = 56°, C = 89°, b = 19$?

3) How many triangles if $a = 7, b = 7, and\ c = 14$?

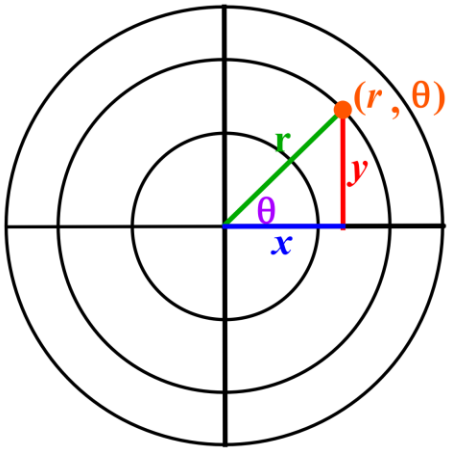
4) What is the principal Values for Tangent?

5) What is the $cos\left(-\frac{\pi}{6}\right)$? $sin\left(-\frac{\pi}{6}\right)$?

Changing from Polar to Rectangular:

$x =$ _____

$y =$ _____



Changing from Rectangular to Polar:

$r =$ _____

$\theta =$ _____

Make sure that your calculator is in _____ mode when using radians and _____ when you have degrees.

Change each point in polar form to rectangular form (coordinates).

1) $\left(6, \frac{\pi}{4}\right)$

2) $\left(-14, -\frac{\pi}{6}\right)$

a) $\left(0, \frac{\pi}{3}\right)$

3) $(4,1.2)$ **4)** $(22, 67°)$ **b)** $(-2.3, -178°)$

Changing from Rectangular to Polar:

So r = _____ and = _____ but add _____ when the point is in _____ or _____

Draw a picture so you know what quadrant the point should be in and stay in.

Change each point in rectangular form to polar form (coordinates).

5) $(2,2)$ **6)** $(-4, -4)$ **7)** $(0,7\ )$

8) $(4, -7)$ **c)** $(-3, -4)$ **d)** $(-8,0)$

Day 3 – Section 6.3B – Converting Equations to Polar and Rectangular Forms

Objectives: SWBAT Convert equations from Polar form to Rectangular Form and visa versa.

Review Questions of the Day:

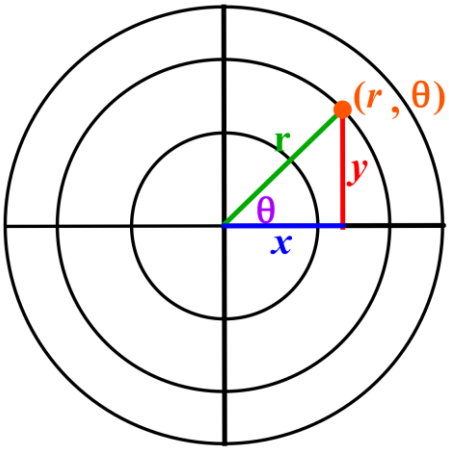
1) What is $csc(0)$?

2) How many triangles are possible if $A = 75°, B = 34°, and\ c = 14.5$?

3) Write the point (1,1) four different ways in polar form.

4) Simplify $4sin^2(x) + 4cos^2(x)$

Rectangular to Polar:



$x^2 + y^2 =$ _____

$\theta = tan^{-1}\left(\frac{\quad}{\quad}\right)$ **Add ____ when θ is in Quad _____ or _____**

Polar to Rectangular:

$x =$ _____ $y =$ _____

Continue to use these above formulas for today, only use them in equations to change the form of the equations.

Writing Equations Steps:

- **Rewrite any x as _____ and any y as _____.**
- **Solve for _____**
- **Simplify the expression**

Equation	Linear			Circle
Rectangular	$y = constant$ $x = constant$	$y = mx$	$y = mx + b$	$x^2 + y^2 = r^2$
Polar	$r = \frac{a}{sin\theta}$ or $r = csc\theta$ $r = \frac{a}{cos\theta}$ or $r = sec\theta$	$\theta =$ _____	$r =$ _____	$r = \pm a$ $r = asin(\theta)$ $r = acos(\theta)$

Change each from rectangular to polar form.

1) $y = 2$

2) $x = 3$

a) $y = 4$

3) $x^2 + y^2 = 25$

4) $4x^2 + 4y^2 = 16$

b) $x^2 + y^2 = 7$

5) $y = x$

5) $y = 2x$

c) $y = \frac{x}{2}$

6) $y = 3x - 1$

d) $y = 2x - 1$

7) $x^2 + 2x + y^2 = 4y$

Change each from polar to rectangular form. Your answer should have no r's and no θ's.

8) $r = 5$

e) $r = -7$

9) $r = 2cos\theta$

f) $r = 3sin\theta$

10) $r = 5csc\theta$

g) $r = sec\theta$

11) $r^2 = 12rcos\theta$

12) $\theta = 45°$

When given a line that goes through the pole, the slope will always be _____so the rectangular form will always be _____.

12) $\theta = 86°$

13) $r^2 = rcos(\theta) + rsin(\theta)$

14) $r = 4cos(\theta) + 5sin(\theta)$

15) $r = \frac{2}{1+\cos(\theta)}$

Day 4 – Section 6.4 – Graphing Cardioid, Limacons, Circles, and Lines

Objectives: SWBAT Graph Equations in Polar Form: Circles, Lines, Cardioids, and Limacons.

Review Questions of the Day:

1) Convert $(-3,-4)$ to polar (use degrees)

2) What is $cos\left(\frac{\pi}{3}\right)$, $\cos\left(-\frac{\pi}{3}\right)$, $sin\left(\frac{\pi}{3}\right)$, $\sin\left(-\frac{\pi}{3}\right)$?

3) What are the principle values for sine?

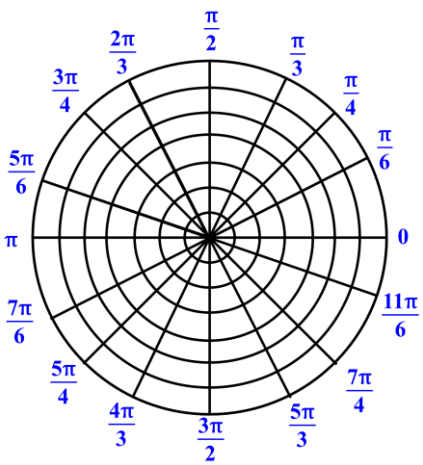
4) What are the principal values for cosine?

Lines:

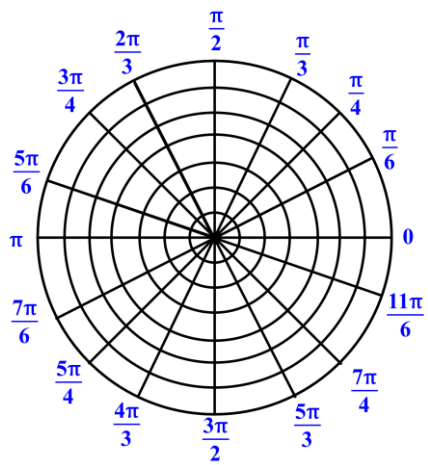
- $\boldsymbol{\theta = angle.}$

Graph the following lines in polar form.

1) $\theta = \frac{\pi}{6}$, $\theta = \frac{-2\pi}{4}$

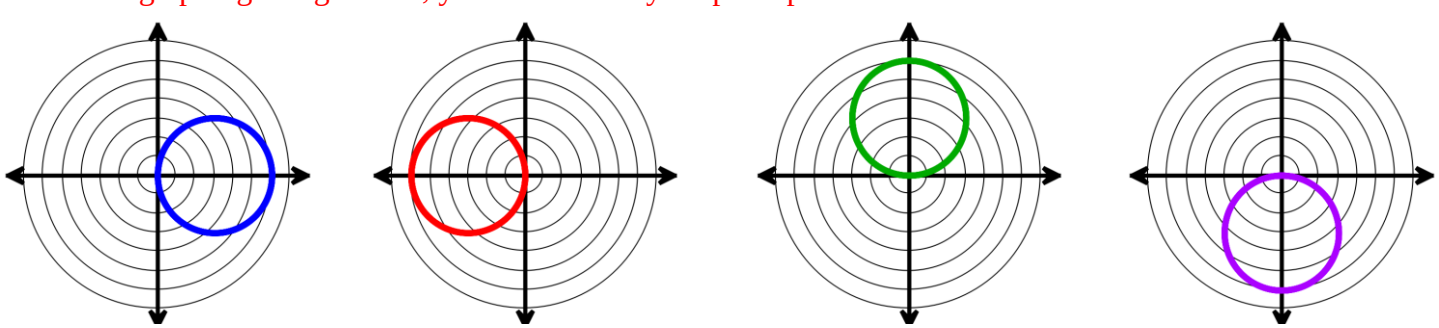


a) $\theta = \frac{3\pi}{4}$, $\theta = \frac{-\pi}{4}$



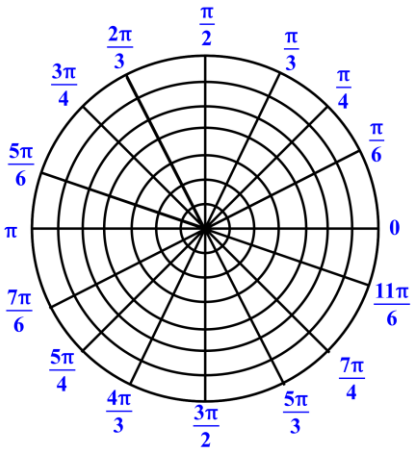
Circles:

- $\boldsymbol{r = m}$ and where m is the radius of a circle with center $(0,0)$
- $\boldsymbol{r = dcos(\theta)}$ or $\boldsymbol{r = dsin(\theta)}$ where d is the diameter of a circle **tangent to the pole.**
- When graphing using a table, you need to use your principle values for sine and cosine.

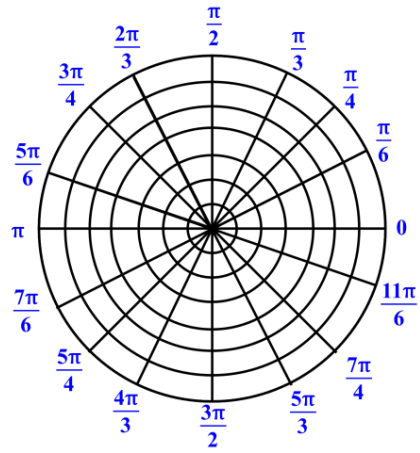


Graph the following circles in polar form.

2) $r = \pm 6$



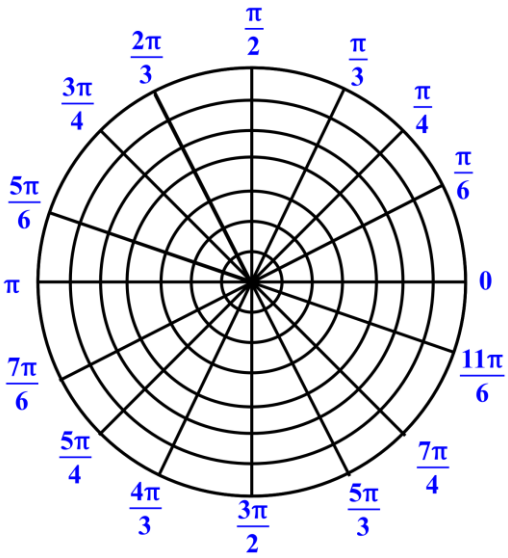
b) $r = \pm 4$



Graph each line or circle on the polar grid.

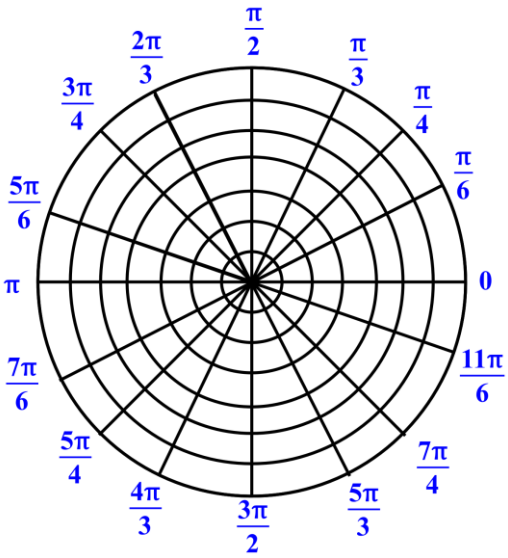
1) $r = 4cos(\theta)$ and $r = -4cos(\theta)$

θ		r
0		
$\frac{\pi}{6}$		
$\frac{\pi}{4}$		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
$\frac{2\pi}{3}$		
$\frac{3\pi}{4}$		
$\frac{5\pi}{6}$		
π		



2) $r = 5sin(\theta)$ and $r = -5sin(\theta)$

θ		r
$\frac{\pi}{2}$		
$\frac{\pi}{3}$		
$\frac{\pi}{4}$		
$\frac{\pi}{6}$		
0		
$\frac{11\pi}{6}$		
$\frac{7\pi}{4}$		
$\frac{5\pi}{3}$		
$\frac{3\pi}{2}$		

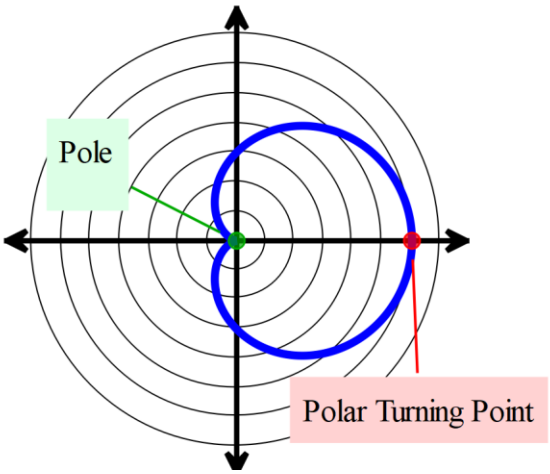


Symmetry:

Since $cos(x) = cos(-x)$ then cosine is _____. Therefore, the graph will have _____ (x axis in rectangular) symmetry. Since the principal values for Cosine exist in Quadrants _____ and _____ we will use angles in Quadrants _____ and _____ as plug-in values.

Likewise, since $sin(-x) = -\sin(x)$ then sine is _____. Therefore the graph will have _____ symmetry. Since the principal values for sine exist in Quadrants _____ & _____ we will use angles in Quadrants _____ and _____ as plug-in values.

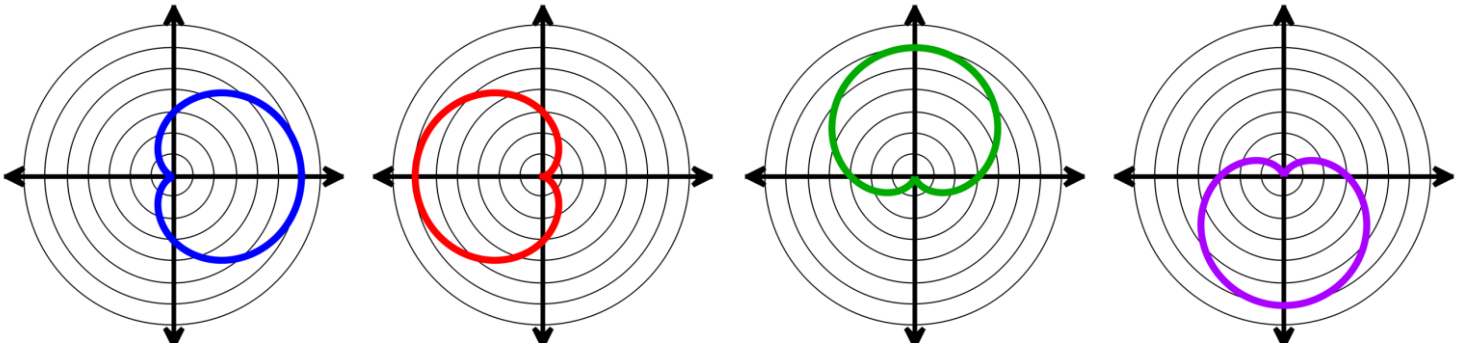
Cardioids:



- Graph of a rolling circle onto another circle of the same radius
- $r = \boldsymbol{m} \pm \boldsymbol{m}cos\theta$ or $r = \boldsymbol{m} \pm \boldsymbol{m}sin\theta$.
- Use your "easy" points based on the Principal Values of Sine and Cosine
  - Sine $\left\{\frac{3\pi}{2}, \frac{11\pi}{6}, 0, \frac{\pi}{6}, \frac{\pi}{2}\right\}$,
  - Cosine $\left\{0, \frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \pi\right\}$
- Cardioids always go through the pole.

https://en.wikipedia.org/wiki/Cardioid#/media/File:Cardiod_animation.gif

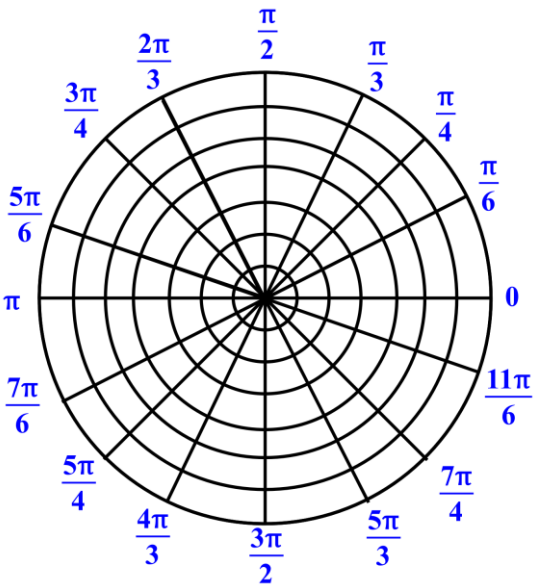
- Cardioids can face in four direction based on the sign and trig function



Graph each cardioids on the polar grid.

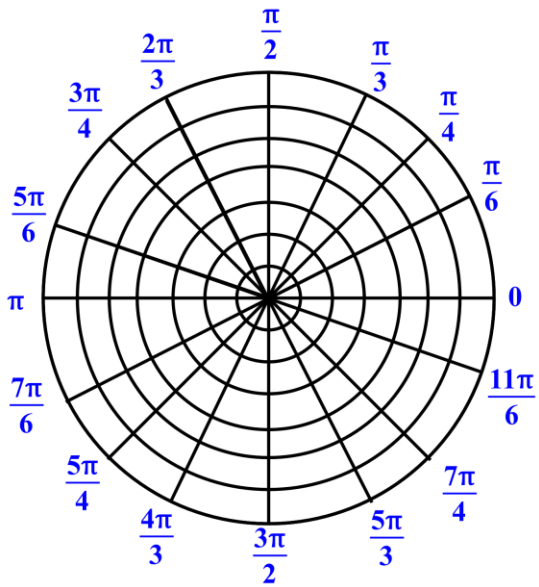
5) $r = 2 - 2cos\theta$

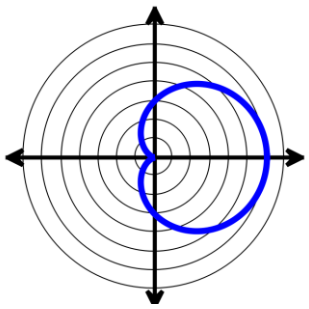
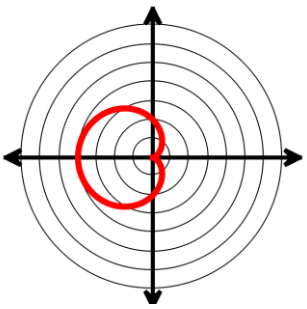
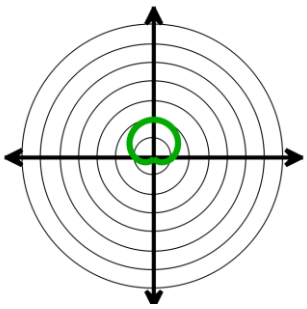
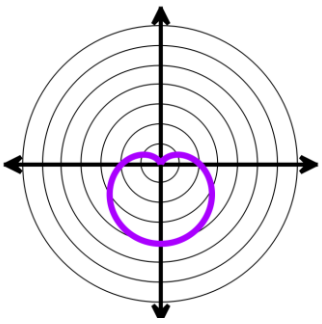
θ		r
0		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
$\frac{2\pi}{3}$		
π		



6) $r = 3 + 3sin\theta$

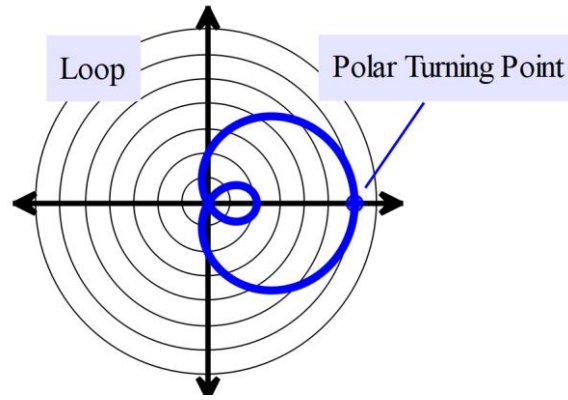
θ		r
$\frac{3\pi}{2}$		
$\frac{11\pi}{6}$		
0		
$\frac{\pi}{6}$		
$\frac{\pi}{2}$		



Direction of Polar Turning Point based on Equation Type			
Cosine		Sine	
Cosine and Positive	Cosine and Negative	Sine and Positive	Sine and Negative
			
Right	Left	Up	Down

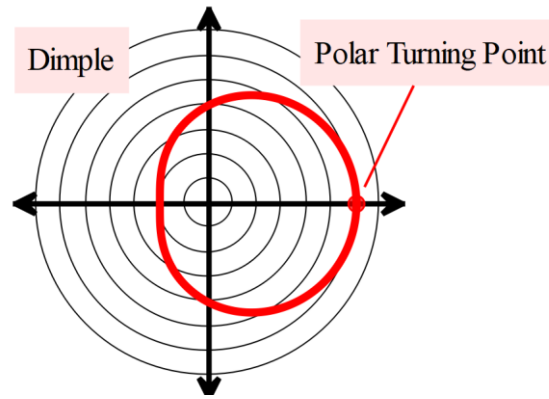
Limaçons:

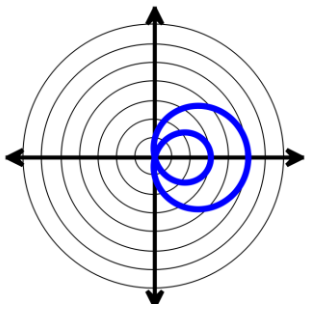
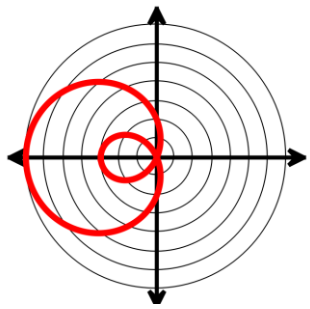
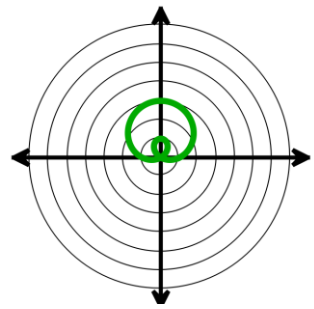
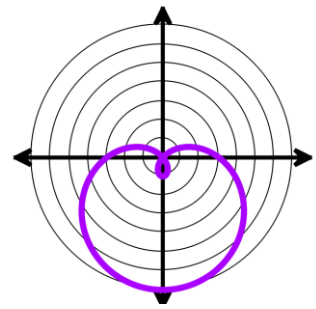
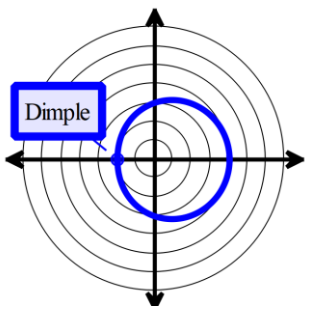
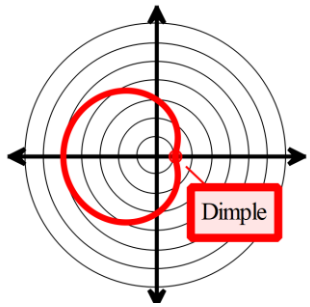
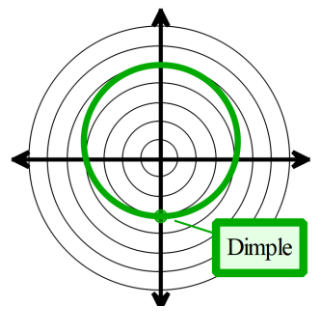
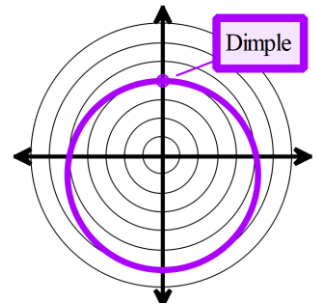
- Graph of a rolling circle onto another circle of the same radius however you values *k* and *m* are not equal
- $r = k \pm m\cos\theta$ or $r = k \pm m\sin\theta$
- Limacons follow the same directional rules as circles and cardioids with respect to sine and cosine.



Loop vs. Dimple Graph

- **A loop** occurs when $k < m$, which basically means when *r* can equal 0.
- Limacons with a LOOP go through the pole and Limacons with a dimple go around the pole.



Direction of Polar Turning Point based on Equation Type			
Cosine		Sine	
Cosine and Positive	Cosine and Negative	Sine and Positive	Sine and Negative
			
			
Right	**Left**	**Up**	**Down**

Given the following equations, decide if the Limacons has a loop or not, and decide which direction the tip points.

7) $r = 7 + 8cos\theta$

Direction:_____

Loop or Dimple:_____

8) $r = 2 - sin\theta$

Direction:_____

Loop or Dimple:_____

c) $r = 9 - 2cos\theta$

Direction:_____

Loop or Dimple:_____

d) $r = 11 - 15sin\theta$

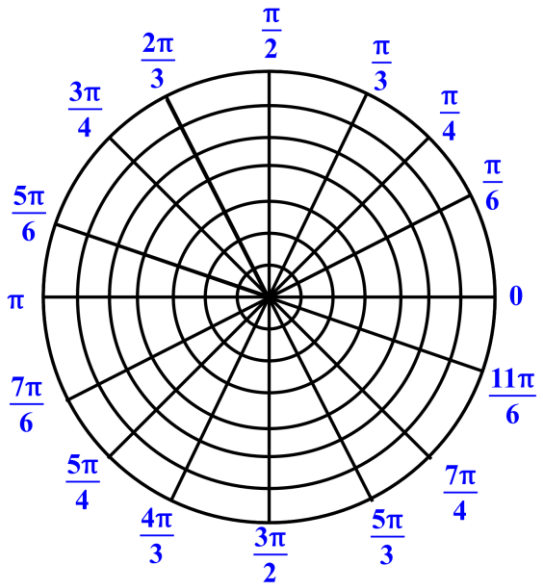
Direction:_____

Loop or Dimple:_____

Use five points for Limacons:

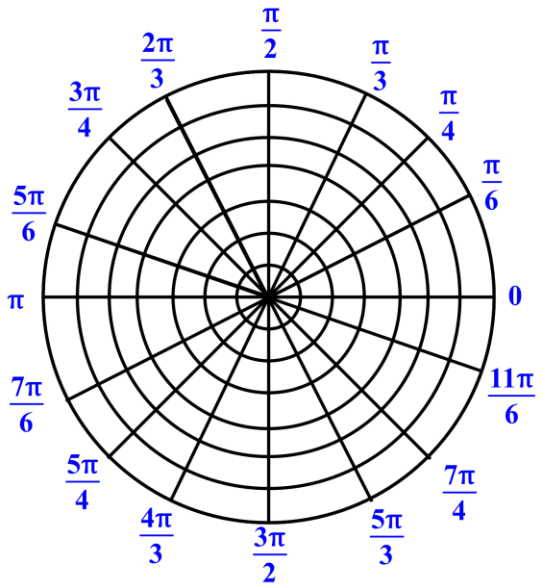
9) $r = 2 + sin\theta$

θ		r
$\frac{3\pi}{2}$		
$\frac{11\pi}{6}$		
0		
$\frac{\pi}{6}$		
$\frac{\pi}{2}$		

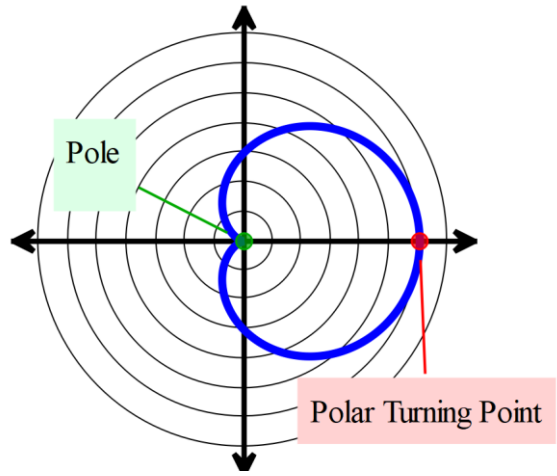


10) $r = 2 - 3cos\theta$

θ		r
0		
$\frac{\pi}{3}$		
$\frac{\pi}{2}$		
$\frac{2\pi}{3}$		
π		

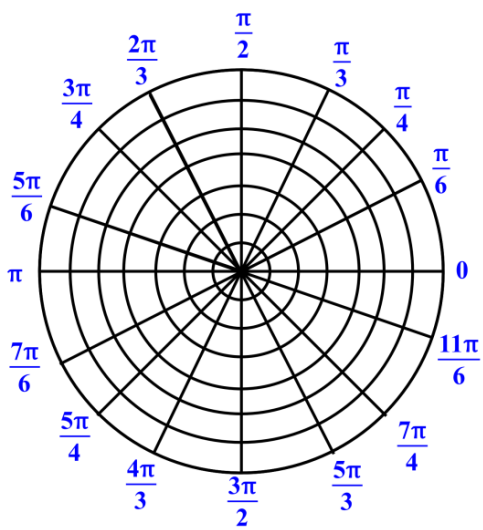


Cardioid Shortcut:

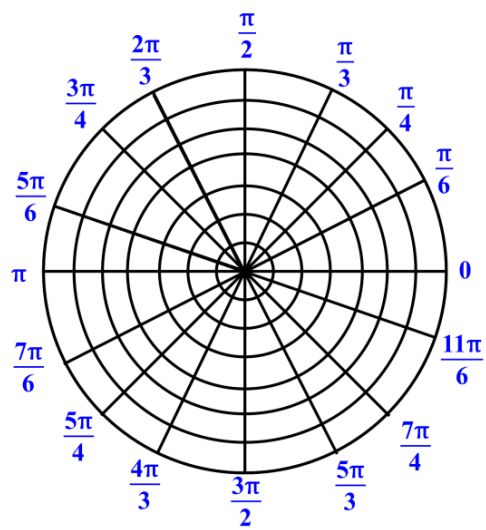


Given the following equations, decide if the Limacons has a loop or not, and decide which direction the tip points.

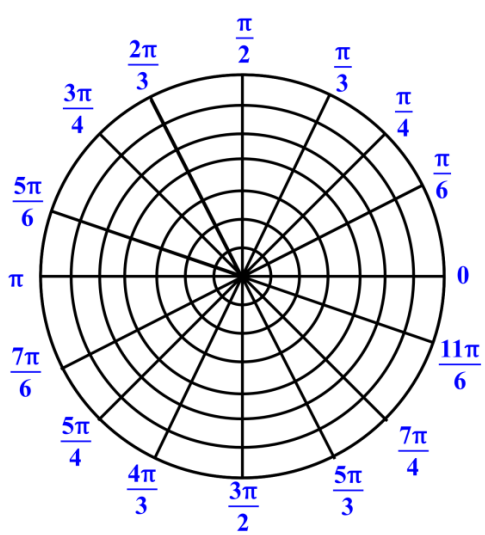
10) $r = 4 - 4cos\theta$



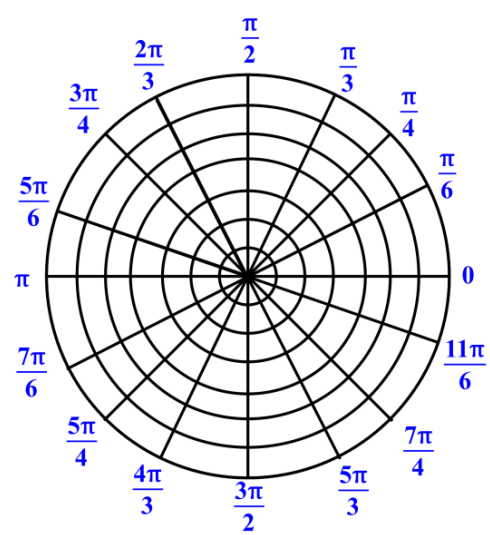
11) $r = 3 + 3sin\theta$



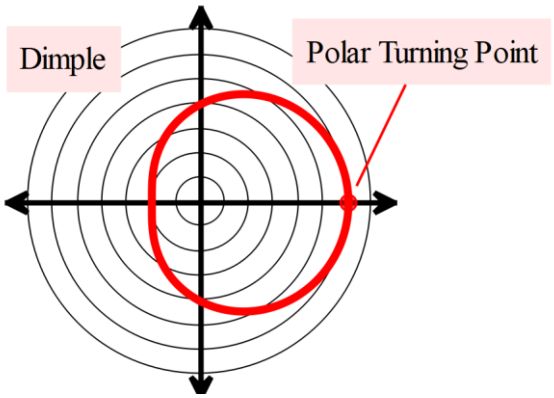
e) $r = 2 + 2cos\theta$

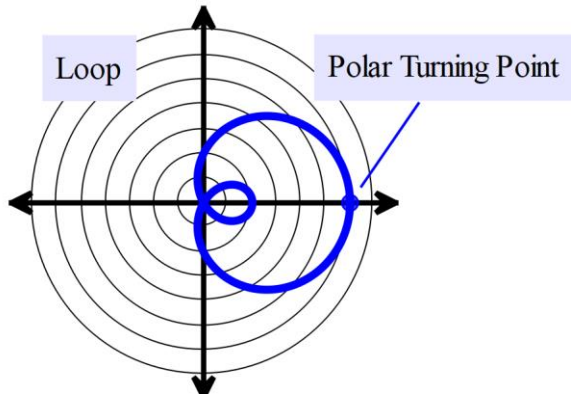


f) $r = 5 - 5sin\theta$



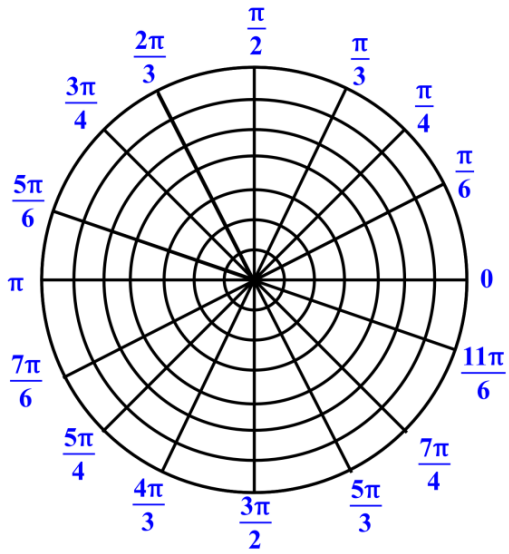
Limaçons Shortcut:



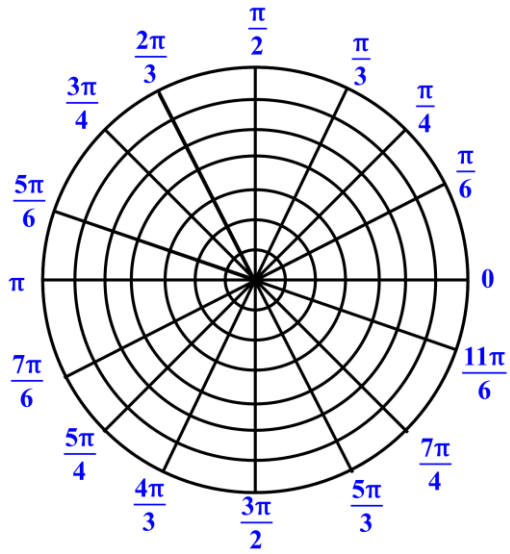


Given the following equations, decide if the Limacons has a loop or not, and decide which direction the tip points.

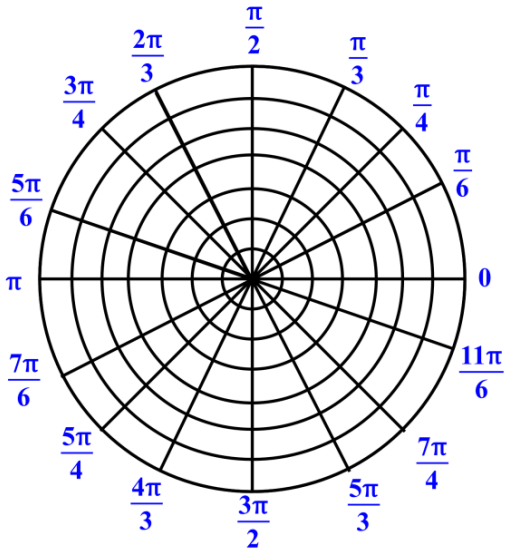
12) $r = 1 - 4cos\theta$



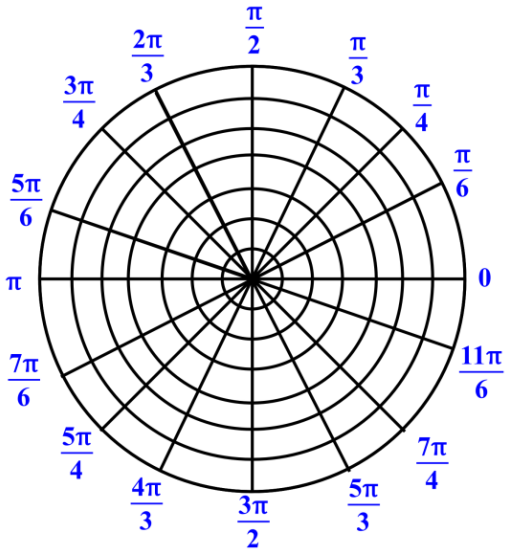
13) $r = 3 + 2sin\theta$



g) $r = 4 - 1cos\theta$



h) $r = 1 - 5sin\theta$



Day 5 – Section 6.4A – Graphing Spirals and Cosine Roses

Objectives: SWBAT Graph Equations in Polar Form: Roses, Lemniscates, and Spirals

Review Questions of the Day:

1) Change $r = 5$ to rectangular form.

2) Change $x = 9$ to polar form.

3) The graph of $r = 8\cos(\theta)$ is a _____ with _____ = 8, and points in _____ direction.

Label each as Line, Circle, Cardioid, or Limaçon.

4) $r = 7$

5) $r = 4 - 4sin(\theta)$

6) $r = 5 + 6cos(\theta)$

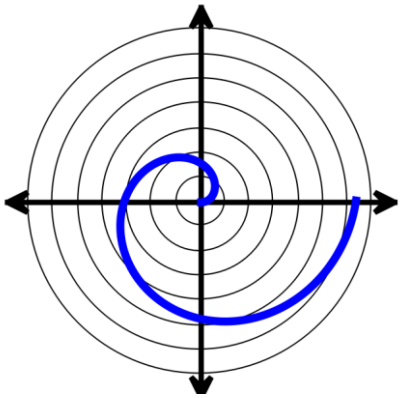
7) $\theta = \frac{\pi}{8}$

8) $rcos(\theta) = 9$

9) $r = 1 + cos(\theta)$

Spirals:

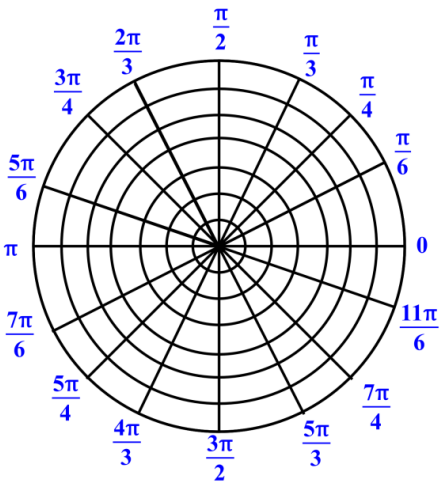
- Spirals look like $r = m\theta$
- The sign of m determines where the spiral begins.
- The direction of a spiral is ALWAYS counterclockwise.



Graph each spiral on the same polar plane using two colors. Use a graphing calculator and the table to get decimal values to the nearest tenth.

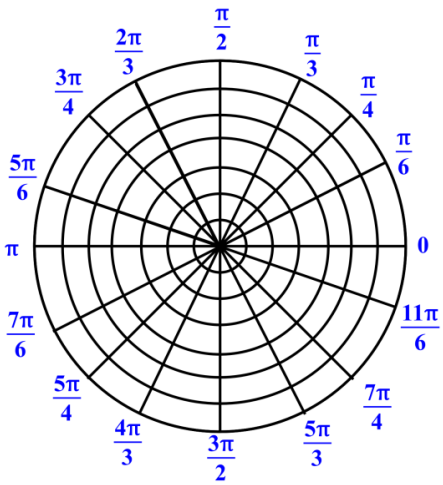
1) $r = \theta$

θ	r
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



2) $r = -2\theta$

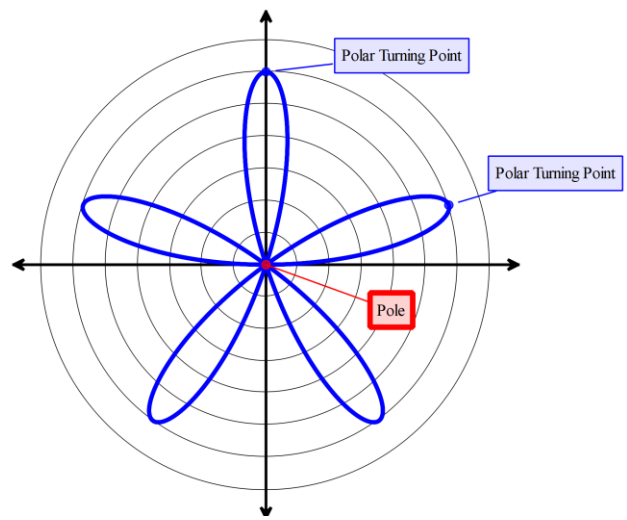
θ	r
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	
2π	



These are graphs with 3 or more petals. Explore these on the calculator to figure out the rule for the number of petals.

a) $r = 6sin(2\theta)$ **b)** $r = 6cos(3\theta)$ **c)** $r = 6sin(5\theta)$ **d)** $r = 6cos(4\theta)$

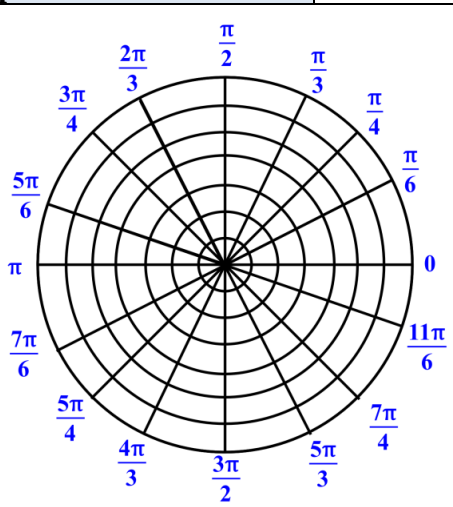
Roses:



- Roses look like $r = msin(n\theta)$ or $r = mcos(n\theta)$
- The length of each petal is m (pole to turning point)
- When n is odd…. There are n number of petals
- When n is even…. There are $2n$ number of petals
- **Cosine Roses** will have horizontal symmetry so there is at least one petal along $\boldsymbol{\theta = 0}$ **(Polar axis).** Which means you start at the _____, and then add $\frac{\pi}{n}$ if n is even, or $\frac{2\pi}{n}$ if n is odd.

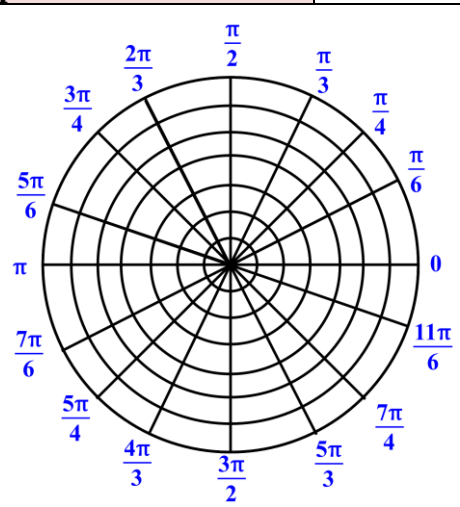
3) $r = 5cos(2\theta)$

Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	
Asymptotes Starting Spot:	



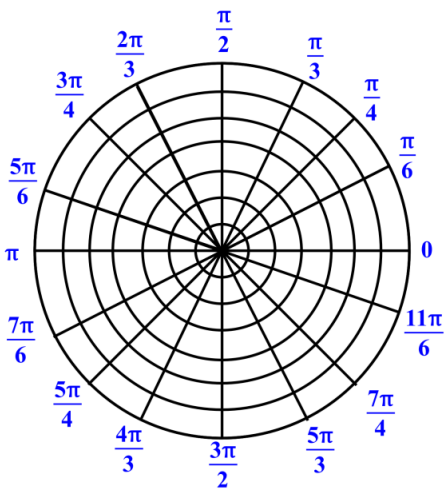
4) $r = -6cos(3\theta)$

Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	
Asymptotes Starting Spot:	



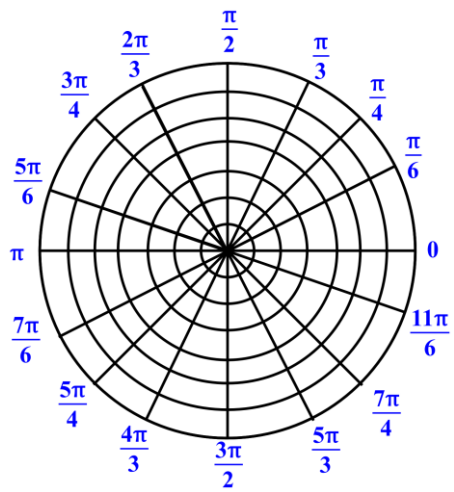
a) $r = 5cos(4\theta)$

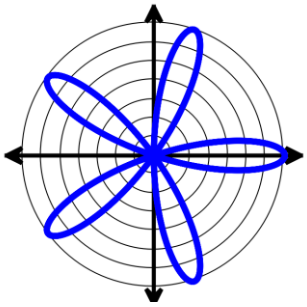
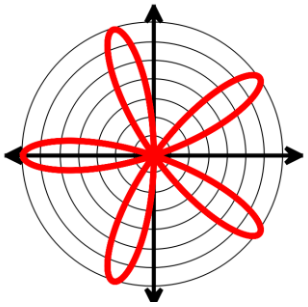
Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	



b) $r = -3cos(5\theta)$

Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	



Direction of the Roses	
Cosine	**– Cosine**
	

Day 6 – Section 6.4B – Graphing All Roses

Objectives: SWBAT Graph Equations in Polar Form: Roses, Lemniscates, and Spirals

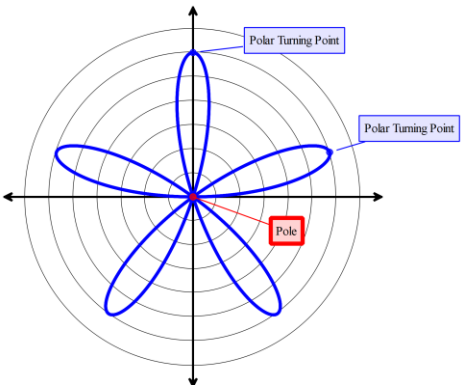
Review Questions of the Day:

Label each as a Cardioid, Line, Circle, Limaçon, Spiral, or Rose.

1) $r = 19cos\theta$ **2)** $r = 2 + 3sin\theta$ **3)** $r = 5cos(5\theta)$ **4)** $rsin\theta = 7$

5) $\theta = 1$ **6)** $r = 3 - 3cos\theta$ 7) $r = 2\theta$ **8)** $r = 2sec\theta$

Graphing Sine Roses:



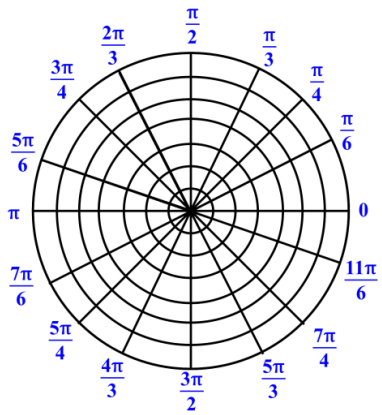
Since a sine rose has vertical symmetry then you must find where the tip of the first petal begins. Since $sin\left(\frac{\pi}{2}\right) = 1$ then you must find where the $angle = \frac{\pi}{2}$ in order to figure out where you have a max or a min.

Where does $sin\theta = 1$? Where does $sin(2\theta) = 1$? Where does $\text{Sin}(3\theta) = 1$?

So $sin(n\theta) = 1$ at $\theta =$ _____ . So a max or a min of a sine rose will occur at $\theta =$ _____ . This is where you will have a tip of one petal always. Then the petals are spread evenly around the polar plane so add _____ each time.

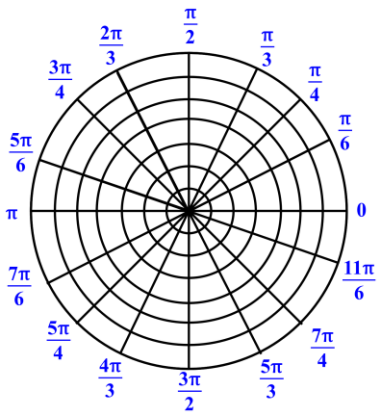
1) $r = 5sin(2\theta)$

Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	
Asymptotes Starting Spot:	



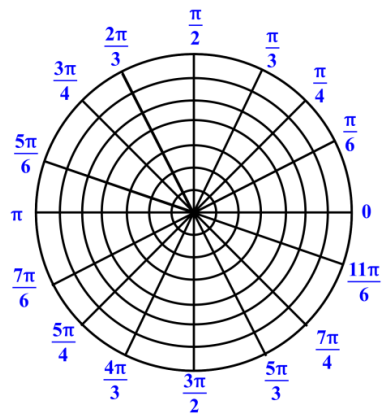
2) $r = 4sin(3\theta)$

Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	
Asymptotes Starting Spot:	



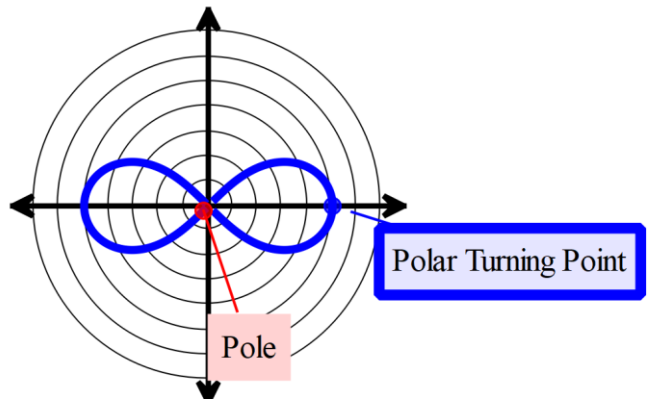
a) $r = 5sin(4\theta)$

Odd or Even:	
Number of petals:	
Starting Spot:	
Distance Between Polar Turning Points:	
Asymptotes Starting Spot:	



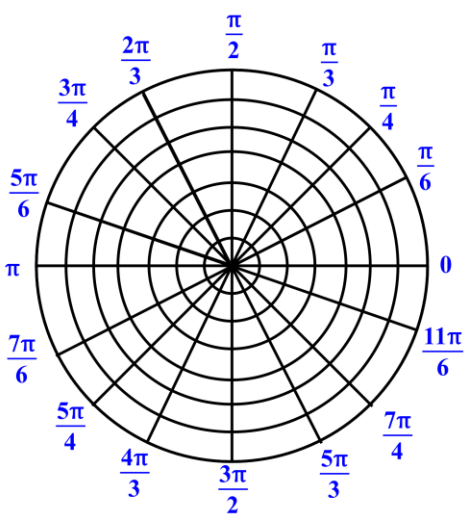
Lemniscates:

- A two petal rose
- $r^2 = m^2 sin(2\theta)$ $\quad or\ r^2 = m^2 \cos(2\theta)$
- **Always two petals, and always** 2θ
- Make sure you use a $\pm$ in calculator

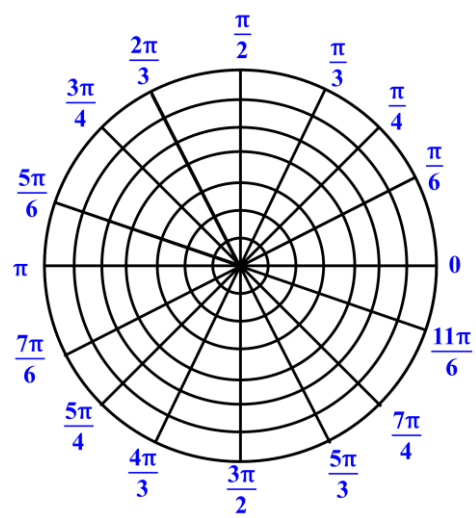


Graph the following.

3) $r^2 = 16sin(2\theta)$



b) $r^2 = 25cos(2\theta)$



Day 7 – Section 6.5 – Polar Form

Objectives: SWBAT Graph Equations in Polar Form: Roses, Lemniscates, and Spirals

Review Questions of the Day:

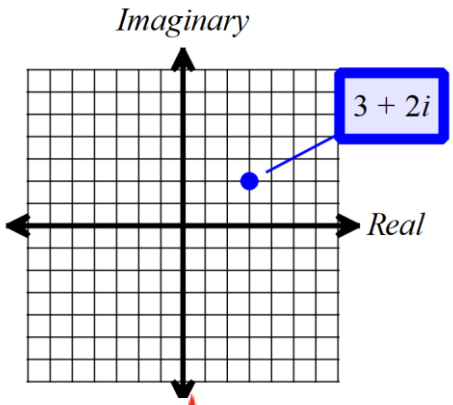
1) Change $r = cos(\theta)$ to rectangular form?

2) Solve for all the values of x $\cos(x) = \frac{1}{2}$

3) What does $i =$, $i^2 =$

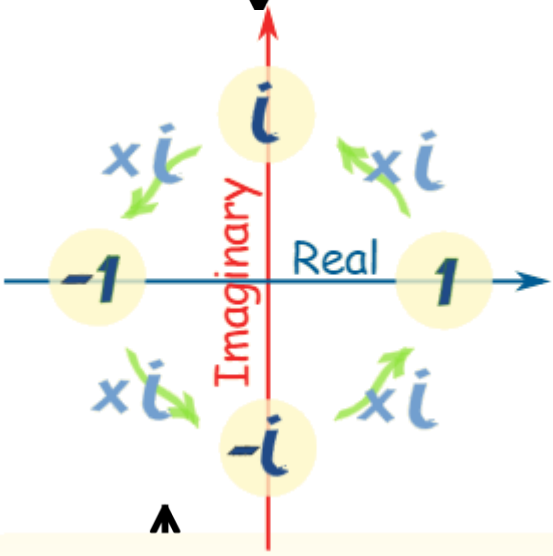
4) Simplify: $\sqrt{-25}$

Imaginary Number:



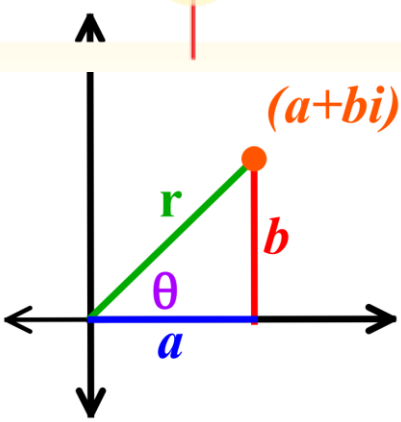
Complex Number:

Complex Plane:



Absolute Value of a Complex Number:

Argument Complex Number:



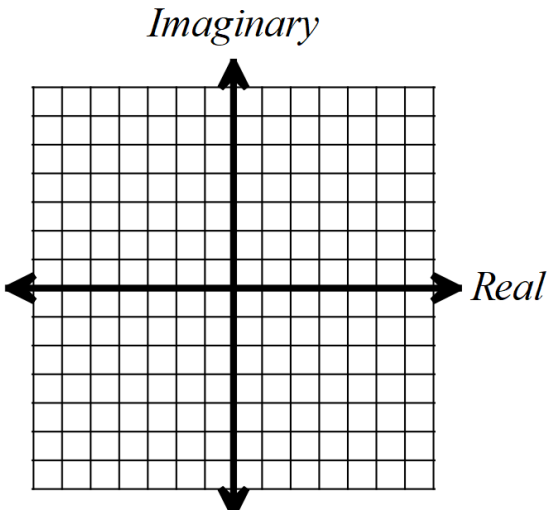
Plot each of the following complex numbers in the complex plane, and find both its absolute value and argument.

1) $3 + 4i$

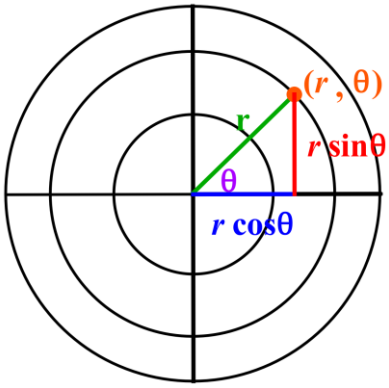
2) $2 - 2i$

3) $3i$

a) $-2 - 4i$



Polar Form of a Complex Number:



Write the following in polar form.

4) $3 + 3i$ **5)** $-4 - 4i$ **b)** $2 - 2i$

Convert from Polar to Rectangular Form: $rcis(\theta)$:

Write the following in rectangular form.

6) $2\big(cos(\pi) + isin(\pi)\big)$

7) $4\big(cos(2.4) + isin(2.4)\big)$

c) $3\left(cos\left(\frac{\pi}{4}\right) + isin\left(\frac{\pi}{4}\right)\right)$

Products and Quotients of Complex Numbers in Polar Form:

Products:

Quotients:

Find the product and leave your answer in the desired form.

8) $z_1 = 6\big(cos(35°) + isin(35°)\big)$, $z_2 = 5\big(cos(55°) + isin(55°)\big)$ Polar Form

9) $\big(6cis(\pi)\big)\left(5cis\left(\frac{\pi}{4}\right)\right)$ Rectangular Form

d) $\big(4cis(17°)\big)\big(3cis(25°)\big)$ Polar Form

Do the following in using Algebra rules, and polar rules. Leave your answer in polar form.

10) $(3-3i)(2+2i)$

Algebra	Polar

Find the quotient and leave your answer in the desired form.

12) $z_1 = 30\big(cos(70°) + isin(70°)\big)$, $z_2 = 5\big(cos(25°) + isin(25°)\big)$ Polar Form

11) $\big(80cis(220°)\big)\big(40cis(300°)\big)$ Rectangular Form

e) $\left(18cis\left(\frac{\pi}{10}\right)\right)\left(2cis\left(\frac{\pi}{20}\right)\right)$ Rectangular

Do the following in using Algebra rules, and polar rules. Leave your answer in polar form.

12) $(2+2i)(1+i)$

Algebra	Polar

Day 8 – Section 6.5A – Intro to DeMoivre's Theorem

Objectives: SWBAT use DeMoivre's Theorem and find the roots of complex numbers

Review Questions of the Day:

1) Solve for all values of x $tan(x) = 1$

2) The graph of $r = 3 + 3\cos(\theta)$ is a ……

3) Change $x = 3$ to polar form.

4) Expand: $(3x - 5)^3$

Imaginary Number:

Complex Number:

Powers of a Complex Number in Polar:

$\left(r(cos\theta + isin\theta)\right)^2 =$

$\left(r(cos\theta + isin\theta)\right)^3 =$

$\left(r(cos\theta + isin\theta)\right)^4 =$

$\left(r(cos\theta + isin\theta)\right)^5 =$

This is called **DeMoivre's Theorem:**

<u>Find each power and write the answer in rectangular form and polar form.</u>

1) $(1+i)^8$

Polar Form: _____

Rectangular Form: _____

2) $\left[2\left(cos\left(\frac{\pi}{10}\right)+isin\left(\frac{\pi}{10}\right)\right)\right]^5$

Polar Form: _____

Rectangular Form: _____

a) $[3(\cos(100°)+isin(100°))]^3$

Polar Form: _____

Rectangular Form: _____

b) $(1-i)^{10}$

Polar Form: _____

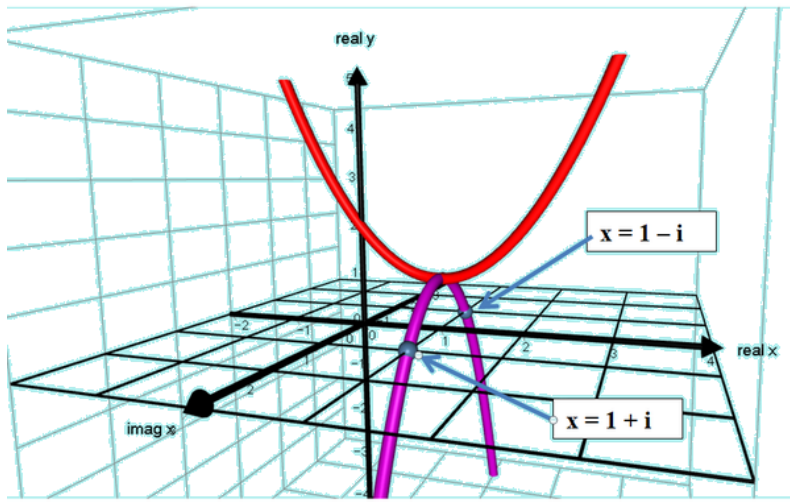
Rectangular Form: _____

DeMoivre's Theorem for Finding Complex Roots:

- All the Answers or Roots are equally spaced throughout the circle
- $Z_{Primary\ Root} = r\left[cos\left(\frac{\theta}{n}\right) + isin\left(\frac{\theta}{n}\right)\right],\ \ n = number\ of\ roots$
- Find the Primary Root then use that to find the Roots of Unity

Complex Root:

Primary Root:



Roots of Unity:

Fundamental Rule of Algebra:

3) Find the complex cube root of $\left(8cos(135°) + isin(135°)\right)$. Write your roots in polar form and in degrees.

4) Find the complex fourth root of $81\left(cos\left(\frac{2\pi}{3}\right)+isin\left(\frac{2\pi}{3}\right)\right)$. Write your roots in rectangular form using radians.

c) Find the complex root of $9\left(cos\left(\frac{3\pi}{2}\right)+isin\left(\frac{3\pi}{2}\right)\right)$. Write your roots in rectangular form using radians.

Day 9 – Section 6.5B – Finding Complex Roots

Objectives: SWBAT use DeMoivre's Theorem and find the roots of complex numbers

Review Questions of the Day:

1) Find the rectangular form of $r = sec\theta$

2) Find the polar form of $y = 7x$

3) Solve the following equation $2x^2 - 32 = 0$

Find all the complex roots. Write the roots in polar form and in rectangular with the argument as an angle between $[0°, 360°]$. Use a calculator to find the rectangular form.

1) Find the complex cube roots of $27(\cos(150°) + isin(150°))$

Polar Form: _____

Rectangular Form: _____

Polar Form: _____

Rectangular Form: _____

2) Find the complex forth roots of $16\left(\cos\left(\frac{\pi}{6}\right) + isin\left(\frac{\pi}{6}\right)\right)$

Polar Form: _____

Rectangular Form: _____

Polar Form: _____

Rectangular Form: _____

a) Find the complex cube roots of $125\left(\cos(\pi) + isin(\pi)\right)$

Polar Form: _____

Rectangular Form: _____

Polar Form: _____

Rectangular Form: _____

Real Solutions:

Imaginary Solutions:

Find all the roots (real and imaginary) for the following equations. Write the answers in polar form, and rectangular form.

3) $x^3 = 27$

4) $x^3 = -64i$

Polar Form: _____

Rectangular Form: _____

Polar Form: _____

Rectangular Form: _____

b) $x^4 = 81$

Polar Form: _____

Rectangular Form: _____

Polar Form: _____

Rectangular Form: _____