Answers to #2 Review

Pre-Calculus Chapter 3 Review & Study Guide Name Key (optional)

Important Generalizations

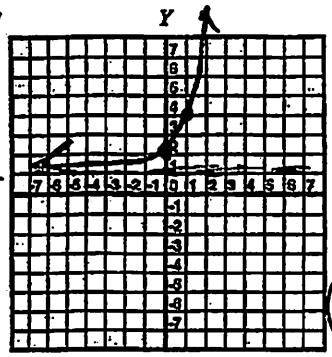
Exp.
$y=b^{x-h}+k$
HA: $y=k$

Log
$y=\log_b(x-h)+k$
VA: $x=h$

$-(x-2)$
right 2

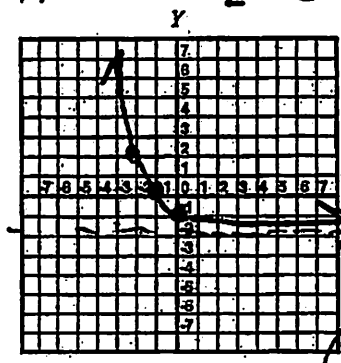
3.1 Exponential & Log Graphs Graph and list domain, range, and asymptotes for each.

1) $y=3^x+1$.



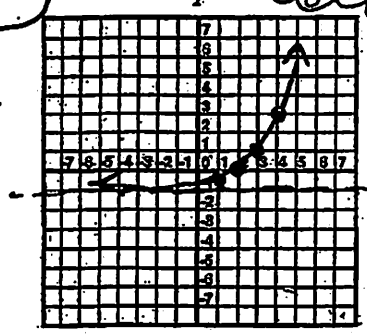
D: $(-\infty,\infty)$
R: $(1,\infty)$
H.A. $y=1$
up 1

2) $y=2^{-x}-2$ $2^{-x}-2$



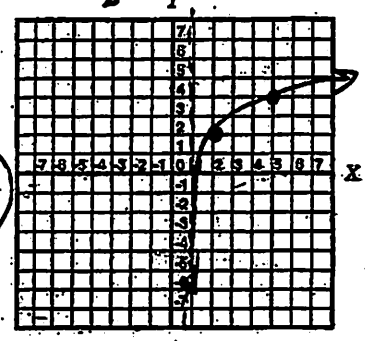
D: $(-\infty,\infty)$
R: $(-2,\infty)$
H.A. $y=-2$
down 2
The $-x$ reflects the graph over "y-axis"

3) $y=(1/2)^{-x+2}-1$



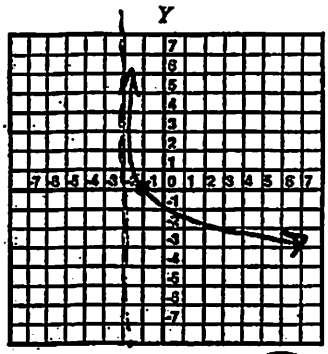
down 1 & reflect
D: $(-\infty,\infty)$
R: $(-1,\infty)$
H.A. $y=-1$
$(2,0)$
$(1,-1/2)$
$(3,1)$
$(4,3)$

4) $y=\log_2(x-1)+2$



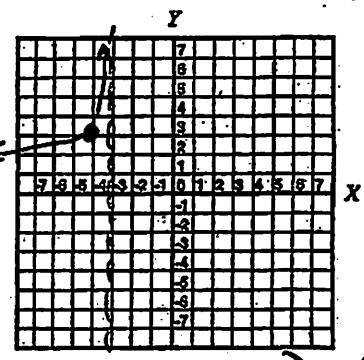
D: $(1,\infty)$
R: $(-\infty,\infty)$
V.A. $x=1$
$(2,2)$
$(5,4)$

5) $y=-\ln(x+2)$



left 2
reflect over y-axis
D: $(-2,\infty)$
R: $(-\infty,\infty)$
V.A. $x=-2$

6) $y=-\log(-x-3)+3$ $=-\log-(x+3)+3$



reflect over "x"
reflect over "y"
left 3, up 3
D: $(-\infty,-3)$
R: $(-\infty,\infty)$
V.A. $x=-3$

Be sure to show a few points on the graph!

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3.2 Solving Exponential Equations

TRY to get the same base. If you can't, then use log or ln.

1) $3^x = 1/27$

$3^x = 3^{-3}$

$x = -3$

2) $4^x = 2$

$2^{2x} = 2^1$

$x = 1/2$

3) $7^{2x} = \sqrt{7}$

$7^{2x} = 7^{1/2}$

$2x = 1/2$

$x = 1/4$

4) $8^{2x-1} = 2^{x+3}$

$2^{3(2x-1)} = 2^{x+3}$

$6x - 3 = x + 3$

$5x = 6$

$x = 6/5$

5) $9^{4x-1} = 3$

$3^{2(4x-1)} = 3^1$

$8x - 2 = 1$

$8x = 3$

$x = 3/8$

6) $5^{x^2} = 50$

$x^2 \log 5 = \log 50$

$x^2 = \frac{\log 50}{\log 5}$

$x = \pm\sqrt{\frac{\log 50}{\log 5}}$

$= \pm 1.56$

7) $7^{2x-1} = 14$

$(2x-1)\log 7 = \log 14$

$2x - 1 = \frac{\log 14}{\log 7}$

$\frac{2x}{2} = \frac{\frac{\log 14}{\log 7} + 1}{2}$

$x \approx 1.1781$

8) $8^{2x-9} = 6^{x+3}$

$(2x-9)\log 8 = (x+3)\log 6$

$2x\log 8 - 9\log 8 = x\log 6 + 3\log 6$

$2x\log 8 - x\log 6 = 9\log 8 + 3\log 6$

$\frac{x(2\log 8 - \log 6)}{(2\log 8 - \log 6)} = \frac{(9\log 8 + 3\log 6)}{(2\log 8 - \log 6)}$

$x \approx 10.1770$

9) $3e^{2x+3} = 5$

$2x + 3 = \ln(5/3)$

$\frac{2x}{2} = \frac{\ln(5/3) - 3}{2}$

$x \approx -1.2446$

Use change of base when solving as well. Change of BASE $\log_b x = \frac{\log x}{\log b}$

10) $3^x = 7$

$\frac{\log 7}{\log 3}$ 1.7712

11) $6^{x+1} = 9$

.2263

$\frac{\log 9}{\log 6} - 1 = x$

12) $7^{2x-3} = 21$

2.2823

$\frac{\log 21}{\log 7} = 2x - 3$

13) $4^{x+2} + 1 = 19$

$4^{x+2} = 18$

$x + 2 = \log 18 / \log 4$

$x \approx .0850$

PROPERTIES of LOGS: 1) logA + logB = $\log AB$ 2) logA – logB = $\log(A/B)$ 3) BlogA = $\log A^B$

Condense:

1) Log2 + log4 + log5

$\log 40$

2) logx – log(x-2) + log(x+5)

$\log \frac{x^2+5x}{x-2}$

3) 2logx + 3logy

$\log(x^2y^3)$

4) lne + ln4 – ln(y-3) + ln(y+3)

$1 + \ln\left(\frac{4y+12}{y-3}\right)$

5) 2lne – ln(x+ 7) + ln(18) –ln(11)

$2 + \ln 18 - \ln(x+7) - \ln 11$

$2 + \ln\left(\frac{18}{11(x+7)}\right)$

Expand:

6) log(9x³ y/w)

$\log 9 + 3\log x + \log y - \log w$

7) log(100xyw/(y-9))³

$3(\log 100 + \log x + \log y + \log w) - 3\log(y-9)$

$3(2 + \log x + \log y + \log w) - 3\log(y-9)$

OR

$6 + 3\log x + 3\log y + 3\log w - 3\log(y-9)$

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LOG EQUATIONS: Isolate the log and then use left to the right = middle OR try and get the equation in the form log = log so you can Cancel the logs.

1) $\log x = 2$ 2) $\ln x = 3$ 3) $\log_4 x = 2$ 4) $\log_x 4 = 2$ 5) $\log(2x-3) = 2$

$10^2 = x$, $x = 100$ | $x = e^3$ | $x = 16$ | $x^2 = 4$, $x = 2$ | $100 = 2x - 3$, $103/2 = x$

We need log(pos #)!

6) $\log_2(4x-1) = \log_2(1)$

$4x - 1 = 1$
$4x = 2$
$x = 1/2$

7) $\log_7(x) + \log_7(x+2) = 1$

$\log_7(x^2+2x) = 1$
$x^2 + 2x = 7$
$x^2 + 2x - 7 = 0$
$x = -2 \pm \sqrt{4 - 4(-7)}/2$
$x = \frac{-2 \pm \sqrt{32}}{2}$
$x = \frac{-2 \pm 4\sqrt{2}}{2}$
$x = -1 \pm 2\sqrt{2}$

Choose the solution that is in domain #7 $\boxed{x = -1 + 2\sqrt{2}}$

8) $\log(x-4) - \log(x+3) = \log 9$

$\log \frac{x-4}{x+3} = \log \frac{9}{1}$
$x - 4 = 9x + 27$
$-31 = 8x$
$x = \frac{-31}{8}$
$\emptyset$

Remember, you must always check your solutions to a log equation in the original since the domain of log(x) is $(0, \infty)$.

* let $x > 0$ & solve *

Find the domain and range of each log function.

1) $\log(6-x) = f(x)$

$6 - x > 0$
$6 > x$
$(-\infty, 6)$

2) $f(x) = 2\log(2x+3)$

$2x + 3 > 0$
$x > -3/2$
$(-3/2, \infty)$

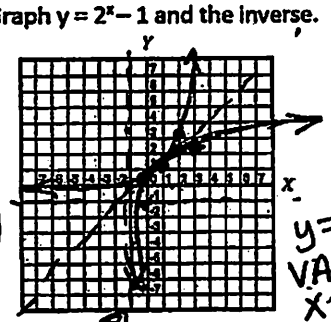
3) $f(x) = \log(x-7)^2$

$+$ $+$ 7 $x \neq 7$

Know how to find and graph an inverse function. Switch x and y and solve for y. In order to graph an inverse function, switch x & y.

1) Graph $y = 2^x - 1$ and the inverse.

DL
Down, left

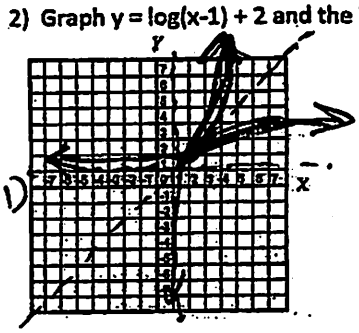


inverse $y = \log_2(x+1)$

$y = \log_2(x+1)$
VA: $x = -1$

2) Graph $y = \log(x-1) + 2$ and the inverse.

RU right, up
UR

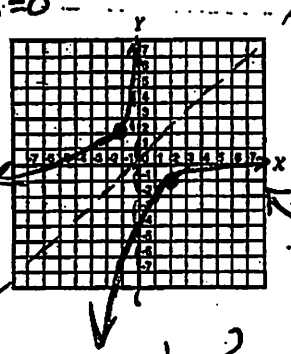


inverse $y = 10^{(x-2)} + 1$

3) Graph $y = -\ln(-x) + 2$ and the inverse.

VA: $x = 0$

RU



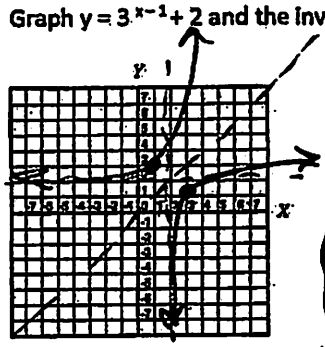
inverse $y = -e^{-(x-2)}$

inverse HA: $y = 0$

sketch

4) Graph $y = 3^{x-1} + 2$ and the inverse.

RU
UR



inverse $y = \log_3(x-2) + 1$

VA: $x = 2$

sketch

Always $y = x$ Symmetry between a function & its inverse!

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Evaluate logs without the use of a calculator.

1) $\log_2 8$ (3) 2) $\log_3 1$ (0) 3) $\log_4 \frac{1}{4}$ (−1) 4) $\log_4(-4)$ (Ø) 5) $\log .000001$ (−6)

6) $\ln e^7$ (7) 7) $\log_7 7^4$ (4) 8) $\log 10^{2x}$ (2x) 9) $e^{\ln 18}$ (18) 10) $\log_2 1 + \log 100$ (2)

11) $\log_8 2$ (1/3) 12) $\log_9 9^{x+1}$ (x+1) 13) $10^{2\log x}$ (x^2) 14) $\log_5 \sqrt[3]{5}$ (1/3) 15) $\ln(1/e^7) + \log_7(\cos 0)$ (−7) (0)

Exponential Growth and Decay: Be able to solve problems involving exponential models as well as logistic growth models. Also, be able to use Newton's Law of Cooling.

Exp. Model $A = A_0 e^{kt}$ Logistic Model $f(x) = a/(b + ce^x)$ → not on test

Newton's Law of Cooling $(A - t_r) = (A_0 - t_r)e^{kt}$

1) Kylee deposits $9,800 into an account earing 4.35% annual interest. How long will it take her money to double? To triple? When will she have $50,000? Assume continuous compounding.

* $\ln 2/.0435$ / * $\ln 3/.0435$ / $\ln\left(\frac{50000}{9800}\right)/.0435$ - 37.5yrs

2) A fossil contains 35% of its original amount of carbon-14. How old is the fossil? Use 5715 years as the half-life.

(c) Should be stored $t = \frac{\ln(.35)}{c} \approx$ 8655.8 yrs old

3) A certain city follows the exponential law. In 1999, there were 34,456 people and in 2011, there were 45,555 people. How many people should we expect in the year 2020?

store $k = \frac{\ln\left(\frac{45,555}{34,456}\right)}{12}$ ↳ Looking for A

$A = 34,456e^{(k)21}$

$A \approx 56168$ people

4) Marilynn wants to have $400,000 in the bank by the time she is 30. How much should she deposit in order to reach this goal in 12 years at 5.567% annual interest? Assume continuous compounding.

$\frac{400,000}{e^{.05567(12)}} = \frac{Pe^{.05567(12)}}{e^{.05567(12)}}$ $P = \$205,085.00$

Challenge type problems:

① $2^{2x} - 3 \cdot 2^x + 2 = 0$

$(2^x - 2)(2^x - 1) = 0$

$2^x = 2$ $2^x = 1$

$x = 1$ $x = 0$

② $e^{2x} - 5e^x + 4 = 0$

$(e^x - 4)(e^x - 1) = 0$

$e^x - 4 = 0$ $e^x - 1 = 0$

$x = \ln 4$ $e^x = 1$ $x = 0$

$x \approx 1.3863$