

## 7.3 HF Answers

Monday, April 23, 2018 11:58 AM

$$9. \frac{x}{(x-3)(x-2)} = \frac{A}{x-3} + \frac{B}{x-2}$$

Multiply both sides of the equation by the least common denominator  $(x-3)(x-2)$  and divide out common factors.

$$x = A(x-2) + B(x-3)$$

$$x = Ax - 2A + Bx - 3B$$

$$x = Ax + Bx - 2A - 3B$$

$$x = (A+B)x - (2A+3B)$$

Equate coefficients of like powers of  $x$ , and equate constant terms.

$$A+B=1$$

$$2A+3B=0$$

Solving the above system for  $A$  and  $B$ , we find  $A=3$  and  $B=-2$ .

$$\frac{x}{(x-3)(x-2)} = \frac{3}{x-3} - \frac{2}{x-2}$$

$$11. \frac{3x+50}{(x-9)(x+2)} = \frac{A}{x-9} + \frac{B}{x+2}$$

Multiply both sides of the equation by the least common denominator  $(x-9)(x+2)$  and divide out common factors.

$$3x+50 = A(x+2) + B(x-9)$$

$$3x+50 = Ax + 2A + Bx - 9B$$

$$3x+50 = Ax + Bx + 2A - 9B$$

$$3x+50 = (A+B)x + (2A-9B)$$

Equate coefficients of like powers of  $x$ , and equate constant terms.

$$A+B=3$$

$$2A-9B=50$$

Solving the above system for  $A$  and  $B$ , we find  $A=7$  and  $B=-4$ .

$$\frac{3x+50}{(x-9)(x+2)} = \frac{7}{x-9} - \frac{4}{x+2}$$

$$13. \frac{7x-4}{x^2-x-12} = \frac{7x-4}{(x-4)(x+3)} = \frac{A}{x-4} + \frac{B}{x+3}$$

Multiply both sides of the last equation by the least common denominator  $(x-4)(x+3)$  and divide out common factors.

$$7x-4 = A(x+3) + B(x-4)$$

$$7x-4 = Ax + 3A + Bx - 4B$$

$$10. \frac{1}{x(x-1)} = \frac{A}{x} + \frac{B}{x-1}$$

Multiply both sides of the equation by the least common denominator  $x(x-1)$  and divide out common factors.

$$1 = A(x-1) + Bx$$

$$1 = Ax - A + Bx$$

$$1 = Ax + Bx - A$$

$$1 = x(A+B) - A$$

Equate coefficients of like powers of  $x$  and equate constant terms.

$$A+B=0$$

$$-A=1$$

Solving for  $A$  and  $B$  gives  $A=-1$  and  $B=1$ .

$$\frac{1}{x(x-1)} = -\frac{1}{x} + \frac{1}{x-1}$$

$$12. \frac{5x-1}{(x-2)(x+1)} = \frac{A}{x-2} + \frac{B}{x+1}$$

Multiply both sides of the equation by the common denominator  $(x-2)(x+1)$  and divide out common factors.

$$5x-1 = A(x+1) + B(x-2)$$

$$5x-1 = Ax + A + Bx - 2B$$

$$5x-1 = Ax + Bx + A - 2B$$

$$5x-1 = (A+B)x + (A-2B)$$

Equate coefficients of like powers of  $x$  and equate constant terms.

$$A+B=5$$

$$A-2B=-1$$

$$A+B=5$$

Solving the above system for  $A$  and  $B$  we find  $A=3$  and  $B=2$ .

$$\frac{5x-1}{(x-2)(x+1)} = \frac{3}{x-2} + \frac{2}{x+1}$$

$$14. \frac{9x+21}{x^2+2x-15} = \frac{9x+21}{(x-3)(x+5)} = \frac{A}{x-3} + \frac{B}{x+5}$$

Multiply both sides of the last equation by the common denominator  $(x-3)(x+5)$  and divide out common factors.

$$9x+21 = A(x+5) + B(x-3)$$

$$9x+21 = Ax + 5A + Bx - 3B$$

common factors.

$$7x - 4 = A(x + 3) + B(x - 4)$$

$$7x - 4 = Ax + 3A + Bx - 4B$$

$$7x - 4 = Ax + Bx + 3A - 4B$$

$$7x - 4 = (A + B)x + (3A - 4B)$$

Equate coefficients of like powers of  $x$ , and equate constant terms.

$$A + B = 7$$

$$3A - 4B = -4$$

Solving the above system for  $A$  and  $B$ , we find  $A =$

$$\frac{24}{7} \text{ and } B = \frac{25}{7}.$$

$$\frac{7x - 4}{x^2 - x - 12} = \frac{24}{7(x - 4)} + \frac{25}{7(x + 3)}$$

$$15. \frac{4}{(2x+1)(x-3)} = \frac{A}{2x+1} + \frac{B}{x-3}$$

Multiply both sides of the equation by the least common denominator  $(2x + 1)(x - 3)$  and divide out common factors.

$$4 = A(x - 3) + B(2x + 1)$$

$$4 = Ax - A + 2Bx + B$$

$$4 = (A + 2B)x + (-A + B)$$

Equate coefficients of like powers of  $x$  and equate the constant terms. Solve for  $A$  and  $B$ .

$$A + 2B = 0$$

$$-A + B = 4$$

$$3A + 6B = 0$$

$$-3A + B = 4$$

$$7B = 4$$

$$B = \frac{4}{7}$$

$$A + 2B = 0$$

$$6A - 2B = -8$$

$$7A = -8$$

$$A = -\frac{8}{7}$$

$$\frac{4}{(2x+1)(x-3)} = \frac{-8}{7(2x+1)} + \frac{4}{7(x-3)}$$

$$17. \frac{4x^2 + 13x - 9}{x(x-1)(x+3)} = \frac{A}{x} + \frac{B}{x-1} + \frac{C}{x+3}$$

Multiply both sides of the equation by the least common denominator  $x(x - 1)(x + 3)$  and divide out common factors.

$$4x^2 + 13x - 9 = A(x - 1)(x + 3) + Bx(x + 3) + Cx(x - 1)$$

$$4x^2 + 13x - 9 = A(x^2 - x - 3) + B(x^2 + 3x) + C(x^2 - x)$$

common factors.

$$9x + 21 = A(x + 5) + B(x - 3)$$

$$9x + 21 = Ax + 5A + Bx - 3B$$

$$9x + 21 = Ax + Bx + 5A - 3B$$

$$9x + 21 = (A + B)x + (5A - 3B)$$

Equate coefficients of like powers of  $x$ , and equate constant terms.

$$A + B = 9$$

$$5A - 3B = 21$$

Solving the above system for  $A$  and  $B$  we find  $A = 6$  and  $B = 3$ .

$$\frac{9x + 21}{x^2 + 2x - 15} = \frac{6}{x - 3} + \frac{3}{x + 5}$$

$$16. \frac{x}{(x+3)(x-1)} = \frac{A}{x+3} + \frac{B}{x-1}$$

Multiply both sides of the last equation by the common denominator  $(x - 1)(x + 3)$  and divide out common factors.

$$x = A(x - 1) + B(x + 3)$$

$$x = Ax - A + Bx + 3B$$

$$x = Ax + Bx - A + 3B$$

$$x = (A + B)x - A + 3B$$

Equate coefficients of like powers of  $x$ , and equate constant terms.

$$A + B = 1$$

$$-A + 3B = 0$$

Solving the above system for  $A$  and  $B$  we find  $A = 3/4$  and  $B = 1/4$ .

$$\frac{x}{x^2 + 2x - 3} = \frac{\frac{3}{4}}{x + 3} + \frac{\frac{1}{4}}{x - 1}$$

Multiply the numerators and the denominators of the left side of the equal sign to simplify the complex fractions.

$$\frac{x}{x^2 + 2x - 3} = \frac{3}{4x + 12} + \frac{1}{4x - 4}$$

$$18. \frac{4x^2 - 5x - 15}{x(x+1)(x-5)} = \frac{A}{x} + \frac{B}{x+1} + \frac{C}{x-5}$$

Multiply both sides of the equation by the common denominator  $x(x + 1)(x - 5)$  and divide out common factors.

$$4x^2 - 5x - 15 = A(x + 1)(x - 5) + Bx(x - 5) + Cx(x + 1)$$

$$4x^2 - 5x - 15 = A(x^2 - 4x - 5) + Bx^2 - 5Bx + Cx^2 + Cx$$

common factors.

$$4x^2 + 13x - 9 = A(x-1)(x+3) + Bx(x+3) + Cx(x-1)$$

$$4x^2 + 13x - 9 = A(x^2 + 2x - 3) + Bx^2 + 3Bx + Cx^2 - Cx$$

$$4x^2 + 13x - 9 = Ax^2 + 2Ax - 3A + Bx^2 + 3Bx + Cx^2 - Cx$$

$$4x^2 + 13x - 9 = Ax^2 + Bx^2 + Cx^2 + 2Ax + 3Bx - Cx - 3A$$

$$4x^2 + 13x - 9 = (A + B + C)x^2 + (2A + 3B - C)x - 3A$$

Equate coefficients of like powers of  $x$ , and equate constant terms.

$$A + B + C = 4$$

$$2A + 3B - C = 13$$

$$-3A = -9$$

Solving the above system for  $A$ ,  $B$ , and  $C$ , we find  $A = 3$  and  $B = 2$ , and  $C = -1$ .

$$\frac{4x^2 + 13x - 9}{x(x-1)(x+3)} = \frac{3}{x} + \frac{2}{x-1} - \frac{1}{x+3}$$

factors.

$$4x^2 - 5x - 15 = A(x+1)(x-5) + Bx(x-5) + Cx(x+1)$$

$$4x^2 - 5x - 15 = A(x^2 - 4x - 5) + Bx^2 - 5Bx + Cx^2 + Cx$$

$$4x^2 - 5x - 15 = Ax^2 - 4Ax - 5A + Bx^2 - 5Bx + Cx^2 + Cx$$

$$4x^2 - 5x - 15 = Ax^2 + Bx^2 + Cx^2 - 4Ax - 5Bx + Cx - 5A$$

$$4x^2 - 5x - 15 = (A + B + C)x^2 + (-4A - 5B + C)x - 5A$$

Equate coefficients of like powers of  $x$  and equate constant terms.

$$A + B + C = 4$$

$$-4A - 5B + C = -5$$

$$-5A = -15$$

Solving the above system for  $A$ ,  $B$ ,  $C$ , we find  $A = 3$ ,  $B = -1$ , and  $C = 2$ .

$$\frac{4x^2 - 5x - 15}{x(x+1)(x-5)} = \frac{3}{x} - \frac{1}{x+1} + \frac{2}{x-5}$$