7.3A HF Answers

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21. $\dfrac{6x-11}{(x-1)^2}=\dfrac{A}{x-1}+\dfrac{B}{(x-1)^2}$

Multiply both sides of the equation by the least common denominator $(x-1)^2$ and divide out common factors.

$6x-11=A(x-1)+B$

$6x-11=Ax-A+B$

Equate coefficients of like powers of x, and equate constant terms.

$A=6$

$-A+B=-11$

Since $A=6$, we find that $B=-5$ by substitution.

$\dfrac{6x-11}{(x-1)^2}=\dfrac{6}{x-1}-\dfrac{5}{(x-1)^2}$

22. $\dfrac{x}{(x+1)^2}=\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$

Multiply both sides of the equation by the common denominator $(x+1)^2$ and divide out common factors.

$x=A(x+1)+B$

$x=Ax+A+B$

Equate coefficients of like powers of x and equate constant terms.

$A=1$

$A+B=0$

Since $A=1$, we find that $B=-1$ by substitution.

$\dfrac{x}{(x+1)^2}=\dfrac{1}{x+1}-\dfrac{1}{(x+1)^2}$

25. $\dfrac{x^2+2x+7}{x(x-1)^2}=\dfrac{A}{x}+\dfrac{B}{x-1}+\dfrac{C}{(x-1)^2}$

Multiply both sides of the equation by the least common denominator $x(x-1)^2$ and divide out common factors

$x^2+2x+7=A(x-1)^2+Bx(x-1)+Cx$

$x^2+2x+7=A(x^2-2x+1)+Bx^2-Bx+Cx$

$x^2+2x+7=Ax^2-2Ax+A+Bx^2-Bx+Cx$

$x^2+2x+7=Ax^2+Bx^2-2Ax-Bx+Cx+A$

$x^2+2x+7=(A+B)x^2+(-2A-B+C)x+A$

$A+B=1$

$-2A-B+C=2$

$A=7$

Since $A=7$, we find that $B=-6$ and $C=10$ by substitution. $\dfrac{x^2+2x+7}{x(x-1)^2}=\dfrac{7}{x}-\dfrac{6}{x-1}+\dfrac{10}{(x-1)^2}$

26. $\dfrac{3x^2+49}{x(x+7)^2}=\dfrac{A}{x}+\dfrac{B}{x+7}+\dfrac{C}{(x+7)^2}$

Multiply both sides of the equation by the common denominator $x(x+7)^2$ and divide out common factors.

$3x^2+49=A(x+7)^2+Bx(x+7)+Cx$

$3x^2+49=A(x^2+14x+49)+Bx^2+7Bx+Cx$

$3x^2+49=Ax^2+14Ax+49A+Bx^2+7Bx+Cx$

$3x^2+49=Ax^2+Bx^2+14Ax+7Bx+Cx+49A$

$3x^2+49=(A+B)x^2+(14A+7B+C)x+49A$

Equate coefficients of like powers of x and equate constant terms.

$A+B=3$

$14A+7B+C=0$

$49A=49$

Solving the above system for A, B, and C, we find $A=1$, $B=2$, and $C=-28$.

$\dfrac{3x^2+49}{x(x+7)^2}=\dfrac{1}{x}+\dfrac{2}{x+7}-\dfrac{28}{(x+7)^2}$

Solving the above system for A, B, and C, we find $A = 1$, $B = 2$, and $C = -28$.

$$\frac{3x^2+49}{x(x+7)^2} = \frac{1}{x} + \frac{2}{x+7} - \frac{28}{(x+7)^2}$$

30. $\dfrac{5x^2-9x+19}{(x-4)(x^2+5)} = \dfrac{A}{x-4} + \dfrac{Bx+C}{x^2+5}$

Multiply both sides of the equation by the common denominator $(x-4)(x^2+5)$ and divide out common factors.

$$5x^2-9x+19 = A(x^2+5)+(Bx+C)(x-4)$$
$$5x^2-9x+19 = Ax^2+5A+Bx^2-4Bx+Cx-4C$$
$$5x^2-9x+19 = Ax^2+Bx^2-4Bx+Cx+5A-4C$$
$$5x^2-9x+19 = (A+B)x^2+(-4B+C)x+5A-4C$$

Equate coefficients of like powers of x and equate constant terms.

$$\begin{aligned} A + B &= 5 \\ -4B + C &= -9 \\ 5A - 4C &= 19 \end{aligned}$$

Solving the above system for A, B, and C, we find $A = 3$, $B = 2$, and $C = -1$.

$$\frac{5x^2-9x+19}{(x-4)(x^2+5)} = \frac{3}{x-4} + \frac{2x-1}{x^2+5}$$

31. $\dfrac{5x^2+6x+3}{(x+1)(x^2+2x+2)} = \dfrac{A}{x+1} + \dfrac{Bx+C}{x^2+2x+2}$

Multiply both sides of the equation by the least common denominator $(x+1)(x^2+2x+2)$ and divide out common factors.

$$5x^2+6x+3 = A(x^2+2x+2)+(Bx+C)(x+1)$$
$$5x^2+6x+3 = Ax^2+2Ax+2A+Bx^2+Bx+Cx+C$$
$$5x^2+6x+3 = Ax^2+Bx^2+2Ax+Bx+Cx+2A+C$$
$$5x^2+6x+3 = (A+B)x^2+(2A+B+C)x+2A+C$$

Equate coefficients of like powers of x, and equate constant terms.

$$\begin{aligned} A + B &= 5 \\ 2A + B + C &= 6 \\ 2A + C &= 3 \end{aligned}$$

Solving the above system for A, B, and C, we find $A = 2$, $B = 3$, and $C = -1$.

$$\frac{5x^2+6x+3}{(x+1)(x^2+2x+2)} = \frac{2}{x+1} + \frac{3x-1}{x^2+2x+2}$$

35. $\dfrac{6x^2-x+1}{x^3+x^2+x+1}=\dfrac{6x^2-x+1}{(x+1)(x^2+1)}=\dfrac{A}{x+1}+\dfrac{Bx+C}{x^2+1}$

Multiply both sides of the last equation by the least common denominator $(x+1)(x^2+1)$ and divide out common factors.

$6x^2-x+1=A(x^2+1)+(Bx+C)(x+1)$

$6x^2-x+1=Ax^2+A+Bx^2+Bx+Cx+C$

$6x^2-x+1=Ax^2+Bx^2+Bx+Cx+A+C$

$6x^2-x+1=(A+B)x^2+(B+C)x+A+C$

Equate coefficients of like powers of x, and equate constant terms.

$A+B=6$

$B+C=-1$

$A+C=1$

Solving the above system for A, B, and C, we find $A=4$, $B=2$, and $C=-3$.

$\dfrac{6x^2-x+1}{x^3+x^2+x+1}=\dfrac{4}{x+1}+\dfrac{2x-3}{x^2+1}$