Name Key Date _____

Unit 6 – Solving Trig Equations – Unit Review

C = calculator needed

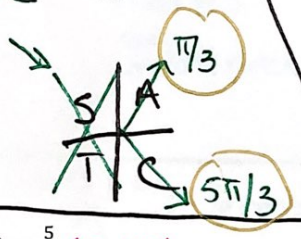
Solve each on the interval $[0, 2\pi)$

1) $cos(x) = \frac{1}{2}$

positive

happens @ 60°

so

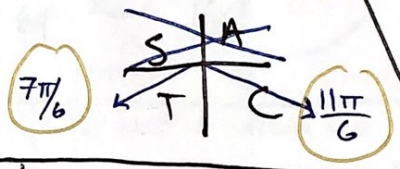


2) $sin(x) = -\frac{1}{2}$

negative

sin=1/2 @ 30°

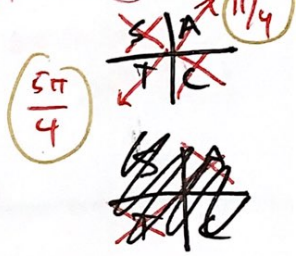
so



3) $tan(x) = 1$

positive ~~negative~~

happens @ 45°



Calculator

C4) $cos(x) = \frac{5}{6}$ ← not on unit circle so use inverse trig

positive

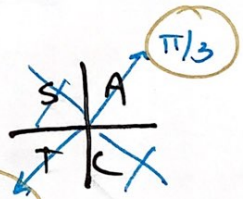
$\cos^{-1}(5/6) = x$

33.56°

5) $tanx = \sqrt{3}$

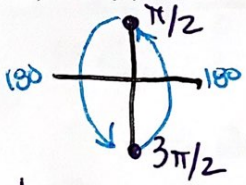
postive

happens @ 60°



Solve for all values of x. Use k = integer.

6) $cos(x) = 0$



happens on y-axis

Mapping = 180° or π

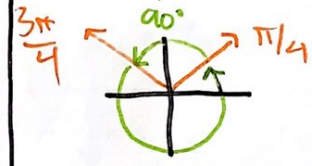
$\boxed{\frac{\pi}{2} + \pi k}$

smallest positive angle

mapping amount

7) $sin(x) = \sqrt{2}/2$

positive



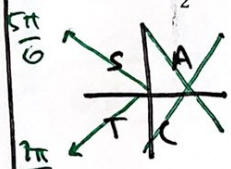
270°

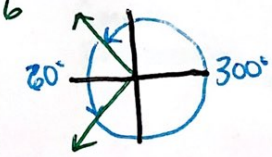
no ≅ mapping so we write individually

$\boxed{\frac{\pi}{4} + 2\pi k \quad \frac{3\pi}{4} + 2\pi k}$

8) $cos(x) = \frac{-\sqrt{3}}{2}$

negative

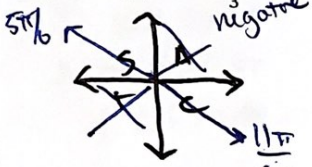


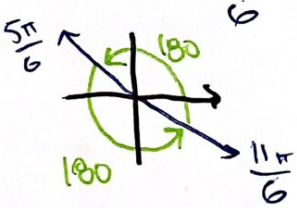


$\boxed{\frac{5\pi}{6} + 2\pi k \quad \& \quad \frac{7\pi}{6} + 2\pi k}$

9) $tan(x) = -\frac{\sqrt{3}}{3}$

negative





≅ Mapping

$\boxed{\frac{5\pi}{6} + \pi k}$

① Factor ② Solve ③ Map not necessary b/c $[0, 2\pi)$

Solve each on the interval $[0, 2\pi)$

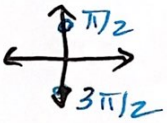
10) $\cos^2 x - 3\cos x = 0$

$\underline{\cos}$

2 terms means GCF

$\cos(\cos - 3) = 0$

$\cos = 0$



$\boxed{\frac{\pi}{2}, \frac{3\pi}{2}}$

$\cos - 3 = 0$

$\cos = 3$

$\cos \neq 3$

so undefined

11) $2\sin^2 x - \sin x - 1 = 0$

3 terms so Factor regularly

$u = \sin(x)$

$2u^2 - 1u - 1$

$-2 \quad -2 \quad 1 \quad -1$

$\frac{2u^2 - 2u}{2u} + \frac{1u - 1}{1}$

$(2u+1)(u-1)$

$2u + 1 = 0$

$2u = -1$

$u = -1/2$

$\sin(x) = -1/2$

$u - 1 = 0$

$u = 1$

$\sin(x) = 1$

$\sin(x) = -1/2$

$\boxed{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\sin(x) = 1$

$\boxed{x = \pi/2}$

12) $\cos^2 x - \sin^2 x = 0$

$\cos^2 x - (1 - \cos^2 x) = 0$

$\cos^2 x - 1 + \cos^2 x = 0$

$2\cos^2 x - 1 = 0$

$u = \cos(x)$

$2u^2 - 1 = 0$

$2u^2 = 1$

$\sqrt{u^2} = \sqrt{1/2}$

$u = \pm\sqrt{\frac{1}{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$

$u = \pm\frac{\sqrt{2}}{2}$

2 terms here is has no GCF so we need to write the equation using just one trig function:

$\sin^2 + \cos^2 x = 1$

$\sin^2 = 1 - \cos^2 x$

$\cos(x) = \pm\frac{\sqrt{2}}{2}$

$\cos(x) = +\frac{\sqrt{2}}{2}$ $\quad \cos = -\frac{\sqrt{2}}{2}$

$\boxed{x = \pi/4, \frac{7\pi}{4} \quad \frac{3\pi}{4}, \frac{5\pi}{4}}$

C13) $2\tan^2 x - 4\tan x = 0$

$u = \tan$

$\frac{2u^2 - 4u}{2u} = 0$

$2u(u - 2) = 0$

$2u = 0$

$u = 2$

$\tan = 0$

$0, \pi$

$u - 2 = 0$

$u = 2$

$\tan = 2$

not on unit circle

$\tan^{-1}(2) = x$

$x = 63.43°$ or 1.107 rads

$\pm 243.43°$ or 4.25 rads

$\boxed{0, 1.107, \pi, 4.25}$

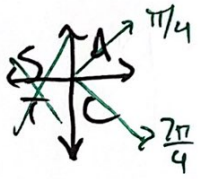
14) $\cos(2x) = \frac{\sqrt{2}}{2}$

$\Delta = 2x$

Double Angle so

$\cos(\Delta) = \frac{\sqrt{2}}{2}$

positive



$\Delta = \frac{\pi}{4} + 2\pi k$

$\frac{2x = \frac{\pi}{4} + 2\pi k}{2}$

$x = \frac{\pi}{8} + \pi k$

$\Delta = \frac{7\pi}{4} + 2\pi k$

$\frac{2x = \frac{7\pi}{4} + 2\pi k}{2}$

$x = \frac{7\pi}{8} + \pi k$

Common Denominator of πk

$\frac{1\pi}{8} + \frac{8\pi}{8} = \frac{9\pi}{8}$

Answer #1 Answer #2

$\frac{7\pi}{8} + \frac{8\pi}{8} = \frac{15\pi}{8}$

Answer #3 Answer #4

$\frac{9\pi}{8} + \frac{8\pi}{8} = \cancel{\frac{17\pi}{8}}$

over 2π so no good

$\boxed{\frac{\pi}{8}, \frac{7\pi}{8}, \frac{9\pi}{8}, \frac{15\pi}{8}}$

Solve on the interval $[0, 2\pi)$

15) $sin(2x) = -1/2$

negative

Sin $\Delta = -1/2$

no mapping here

$\Delta = \frac{7\pi}{6} + 2\pi k$ $\quad$ $\Delta = \frac{11\pi}{6} + 2\pi k$

$\frac{2x = \frac{7\pi}{6} + 2\pi k}{2}$ $\quad$ $2x = \frac{11\pi}{6} + 2\pi k$

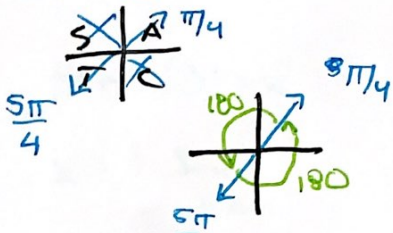
$x = \frac{7\pi}{12} + \pi k$ $\quad$ $x = \frac{11\pi}{12} + \pi k$

$\frac{7\pi}{12} + \frac{12\pi}{12} = \frac{19\pi}{12}$ $\quad$ $\frac{11\pi}{12} + \frac{12\pi}{12} = \frac{23\pi}{12}$

16) $tan(3x) = 1$

Pos

tan $\Delta = 1$



$\Delta = \frac{\pi}{4} + \pi k$

$\frac{3x = \frac{\pi}{4} + \pi k}{3}$

$x = \frac{\pi}{12} + \frac{\pi}{3}k$

$\frac{\pi}{12} + \frac{4\pi}{12} = \frac{5\pi}{12}$

$\frac{5\pi}{12} + \frac{4\pi}{12} = \frac{9\pi}{12} = \frac{3\pi}{4}$

$\frac{9\pi}{12} + \frac{4\pi}{12} = \frac{13\pi}{12}$

$\frac{13\pi}{12} + \frac{4\pi}{12} = \frac{17\pi}{12}$

$\frac{17\pi}{12} + \frac{4\pi}{12} = \frac{21\pi}{12} = \frac{7\pi}{4}$

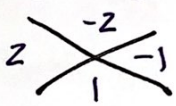
Reduce

Reduce

Solve for all values of x. Use k as an integer.

17) $2cos^2x + cosx - 1 = 0$ $\quad$ $u = \cos x$

$2u^2 + u - 1 = 0$



$\frac{(2u^2+2u)}{2u}\frac{(-1u-1)}{-1}$

$(2u-1)(u+1)$

$2u-1=0$ $\quad$ $u+1=0$

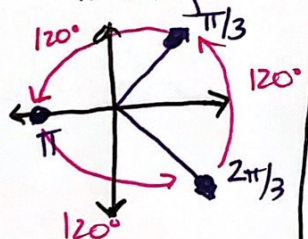
$u = 1/2$ $\quad$ $u = -1$

$\cos(x) = 1/2$

$x = \pi/3, 4\pi/3$

$\cos(x) = -1$

π only



$\cong$ mapping

$\boxed{\frac{\pi}{3} + \frac{2\pi}{3}k}$

18) $sin^2x = sinx$

$\sin^2 x - \sin x = 0$

$u = \sin x$

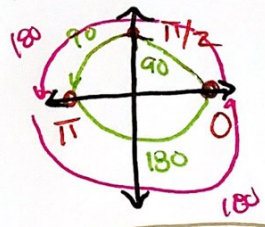
$\frac{u^2 - u = 0}{u}$

$u(u-1) = 0$

$u = 0$ $\quad$ $u - 1 = 0$

$\sin = 0$ $\quad$ $\sin = +1$

$x = 0, \pi$ $\quad$ $x = \pi/2$



no $\cong$ mapping for all 3 so try 2

$\boxed{x = 0 + \pi k}$

$\boxed{x = \frac{\pi}{2} + 2\pi k}$

19) $cos^2x - 1 = 0$

Perfect square

$u = \cos x$

$u^2 - 1$

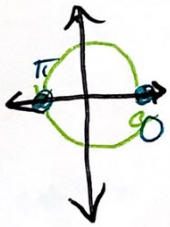
$(u-1)(u+1) = 0$

$u - 1 = 0$ $\quad$ $u + 1 = 0$

$u = 1$ $\quad$ $u = -1$

$\cos = 1$ $\quad$ $\cos = -1$

0 $\quad$ π



$\cong$ mapping

$\boxed{0 + \pi k}$

C20) $2tan^2x - 5tanx - 3 = 0$ Round radians to two decimal places and solve **for all values** of x, using k as an integer.

$u = \tan x$

$2u^2 - 5u - 3 = 0$

-6

-6 $\quad$ 1

-5

$(2u^2 - 6u)(+1u - 3) = 0$

$2u \qquad 1$

$(2u+1)(u-3) = 0$

$2u + 1 = 0 \qquad u - 3 = 0$

$u = -1/2 \qquad u = 3$

$\tan = -1/2 \qquad \tan = 3$

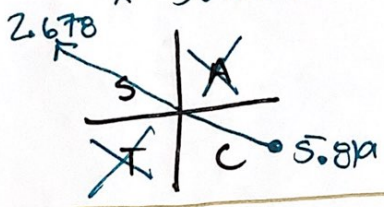
Both not on unit circle

$\tan^{-1}(-1/2) = x$

$x = -0.464$

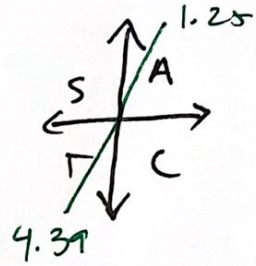
need positive version so add 2π

$x = 5.819$



$\tan^{-1}(3) = x$

$x = 1.25$



$\boxed{x = 2.678 + \pi k \qquad x = 1.25 + \pi k}$

BONUS: $sin^2x - 3sinx + 1 = 0$ Solve for all values of x. Use radian mode.

$u = \sin x$

$u^2 - 3u + 1 = 0$

$a = b \quad b = -3 \quad c = 1$

No Factors $\longrightarrow$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Quadratic Formula

$\dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(1)(1)}}{2(1)}$

$\dfrac{3 \pm \sqrt{9 - 4}}{2}$

$\dfrac{3 \pm \sqrt{5}}{2} \longrightarrow u = \dfrac{3 \pm \sqrt{5}}{2}$

$u = \dfrac{3 + \sqrt{5}}{2}$

$u = 2.618$

$\sin = 2.618 \rightarrow$ undefined

$\sin = \dfrac{+3 - \sqrt{5}}{2} = 0.381$

not on unit circle

$\sin^{-1}(0.381) = x$

$x = .39$ & 2.75